Probabilistic Gas Leak Rate Estimation Using Submodular Function Maximization With Routing Constraints

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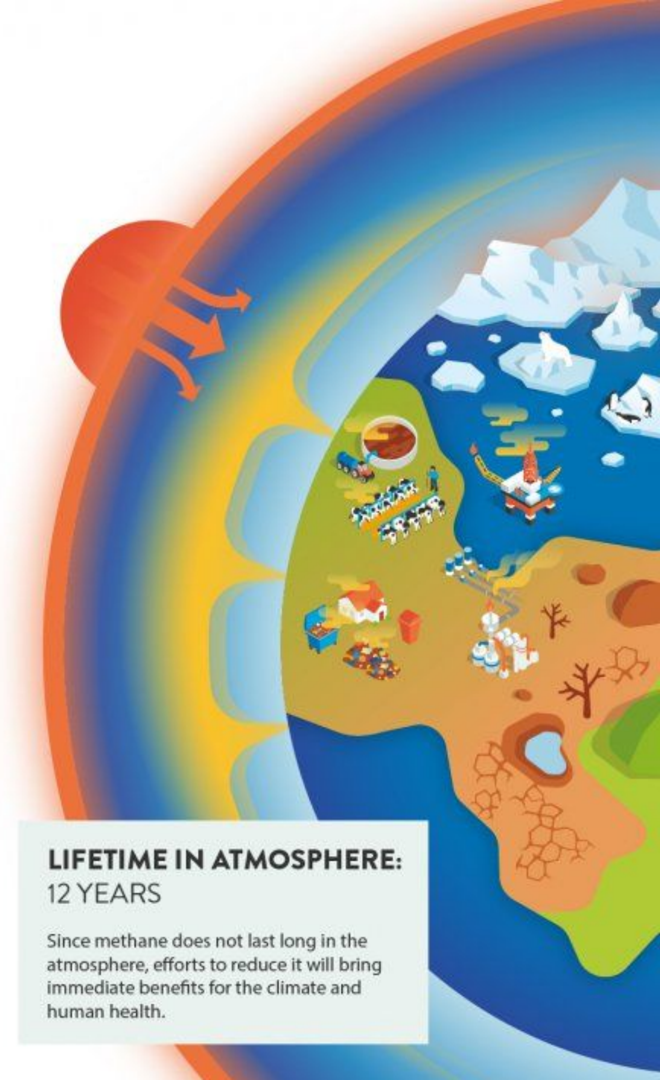
Outline

- Problem
- Assumptions
- Prior approaches
- Method
  - Leak rate estimation
  - Informative Path Planning (IPP)
- Limitations
- Conclusion

Problem

- Monitor fugitive gas leaks
  - Harmful greenhouse gases
- Non-trivial problem
  - Environmental factors
  - Simultaneous leaks
  - Road network constraint
  - Informative Path Planning (IPP) problem
  - Distance budget



https://www.ccacoalition.org/en/slcps/methane (figure source)

Assumptions

- Flat terrain
- Non-overlapping gas plumes
- Constant environmental factors
- Known candidate leak locations
- Threshold leak rates to find source

Permian basin oil fields

https://fortune.com/2018/05/25/photograph-permian-basin-oil/ (figure source)

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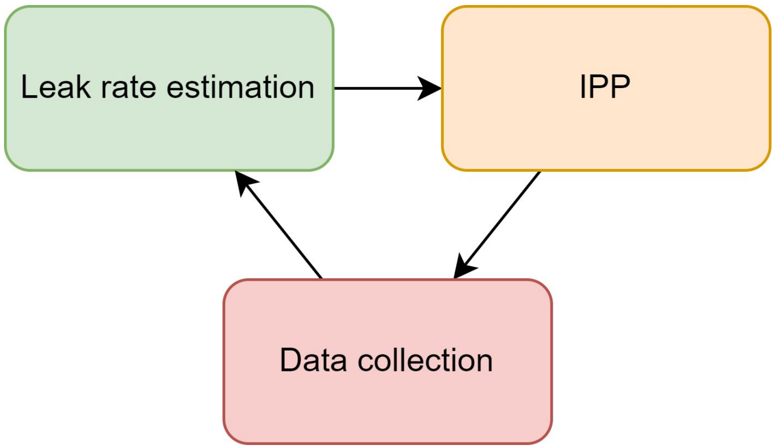
Prior approaches

- Satellites [Pandey et al. 2019]
- Fixed sensor networks [Allen 2020]
- Neural Networks [Travis et al. 2020]
- Bayesian approach [Albertson et al. 2016]
    - Fix prior over leak rate
    - Mutual information for IPP
    - Collect data
    - Calculate posterior of leak rates
    - Repeat until convergence



Prior approaches

- Bayesian approach
  - GEV type II prior
  - Likelihood

$$p(m|s) = \frac{1}{\sigma_e\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{m - C(s,x,y,z)}{\sigma_e}\right)^2\right]$$

  - Gas dispersion model

$$C(s,x,y,z) = \frac{s}{U}\left(\frac{\bar{A}}{\overline{z}(x)} exp\left[-\left(\frac{Bz}{\overline{z}(x)}\right)^2\right]\right) \times \left(\frac{1}{\sqrt{2\pi}\sigma_y} exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\right)$$

Prior approaches

- Bayesian approach
  - Posterior (intractable to an analytical solution)

$$p(s|m) = \frac{p(m|s)p(s)}{\int_{s_1 \epsilon S} p(m|s_1)p(s_1)ds_1}$$

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Method (Gaussian assumption)

- Gaussian prior

$$p(s) \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

- Likelihood (gas dispersion model factorization)

$$C(s,x,y,z) = s \underbrace{\frac{1}{U}\left(\frac{\bar{A}}{\overline{z}(x)} \exp\left[-\left(\frac{Bz}{\overline{z}(x)}\right)^2\right]\right)\left(\frac{1}{\sqrt{2\pi}\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\right)}_{constant,\ independent\ of\ function\ input\ 's'} = s\mathcal{A}(x,y,z)$$

$$\begin{aligned} p(m|s) &= \frac{1}{\sigma_e\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{m - C(s,x,y,z)}{\sigma_e}\right)^2\right] \\ &= \frac{1}{\sigma_e\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{m - s\mathcal{A}(x,y,z)}{\sigma_e}\right)^2\right] \end{aligned}$$

$$p(m|s) \sim \mathcal{N}(s\mathcal{A}, \sigma_e^2)$$

Method (Gaussian assumption)

- Analytical posterior
    - Linear in the number of data samples

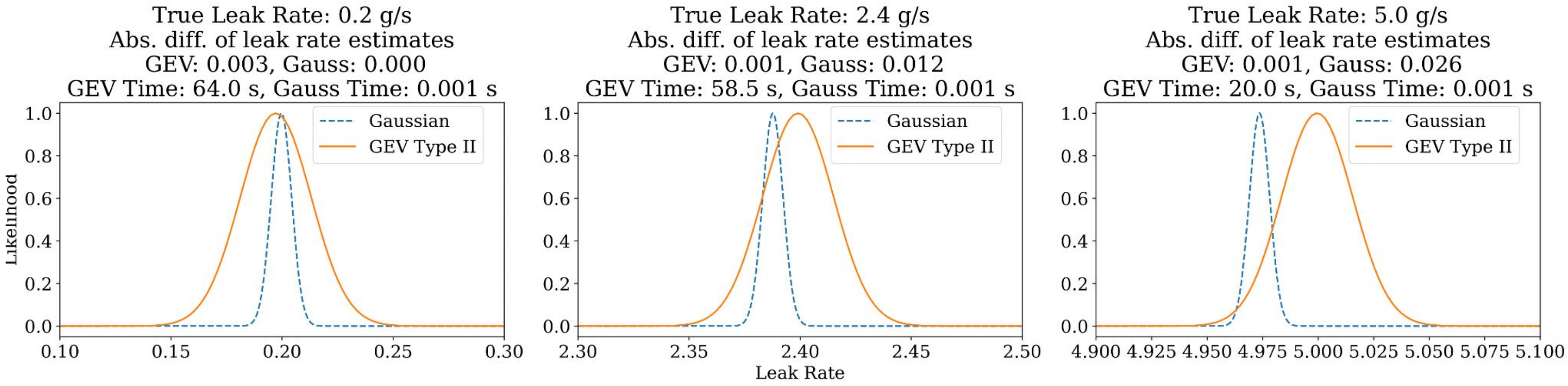
$$p(s|M) \propto \mathcal{N}\left(\frac{M^T\mathbf{A}\sigma_s^2 + \mu_s\sigma_e^2}{\mathbf{A}^T\mathbf{A}\sigma_s^2 + \sigma_e^2}, \frac{\sigma_e^2\sigma_s^2}{\mathbf{A}^T\mathbf{A}\sigma_s^2 + \sigma_e^2}\right)$$

M set of gas concentration data samples collected from the field

$\mathbf{A}$ vector containing the output of the factorized gas dispersion model with unit leak rate

Simulated experimental results

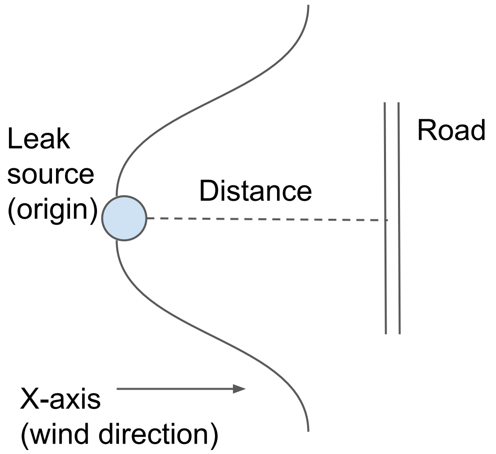
- Gaussian prior convergence results
  - Converges to true leak rate
  - Five orders of magnitude faster



Simulated experimental results

- Predicts correct path information order
  - Five orders of magnitude faster

Distance to oil well along X-axis (meters)	Compute Time (secs)	
	GEV Type II prior	Gaussian prior
0.2	22.42391	0.00017
0.8	16.90775	0.00016
1.4	30.65850	0.00017
2.4	41.51963	0.00021
3.0	42.78649	0.00017
5.0	41.21629	0.00020



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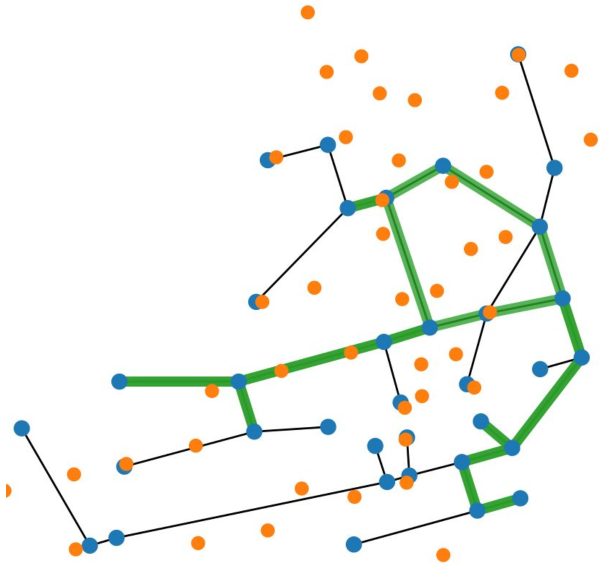
Prior approaches (IPP)

- IPP literature
  - Focuses on scalar fields (e.g., gas concentration)
  - Not directly applicable
  - We plan paths only for estimating leak rates

- Prior approaches considered brute force search
  - Expected entropy reduction (EER)
  - Does not scale to large graphs
  - Exponential number of possible paths
  - Paths might not approach any leak sources



Example road network

Method (IPP)

- EER measures mutual information

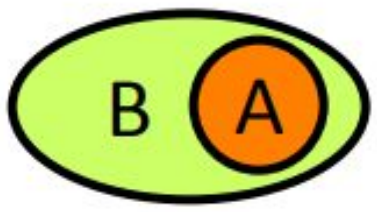
$$\varphi[S;M] = H(S) - \int_{m \epsilon M} H(S|m)p(m)dm$$

  - Submodular function (diminishing returns)

$$f(X \cup \{u\}) - f(X) \geq f(Y \cup \{u\}) - f(Y)$$
$$\forall X \subseteq Y \subset T \text{ and } u \in T \backslash Y$$

Method (IPP)

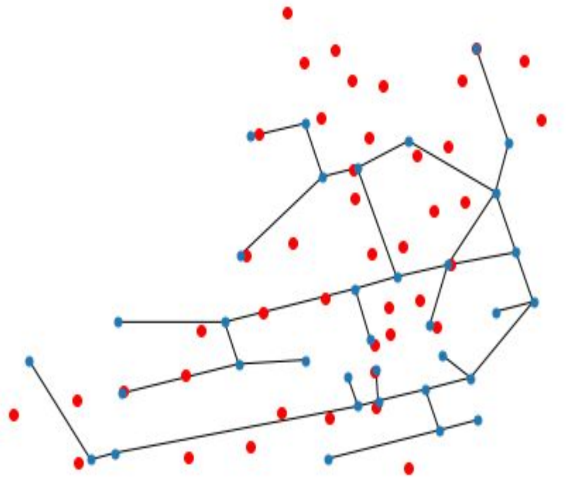
- EER measures mutual information
  - Submodular function (diminishing returns)



http://luthuli.cs.uiuc.edu/~daf/courses/Opt-2019/BilevelSubmodularMaterial/submodularity-slides.pdf (figure source)

Method (IPP)

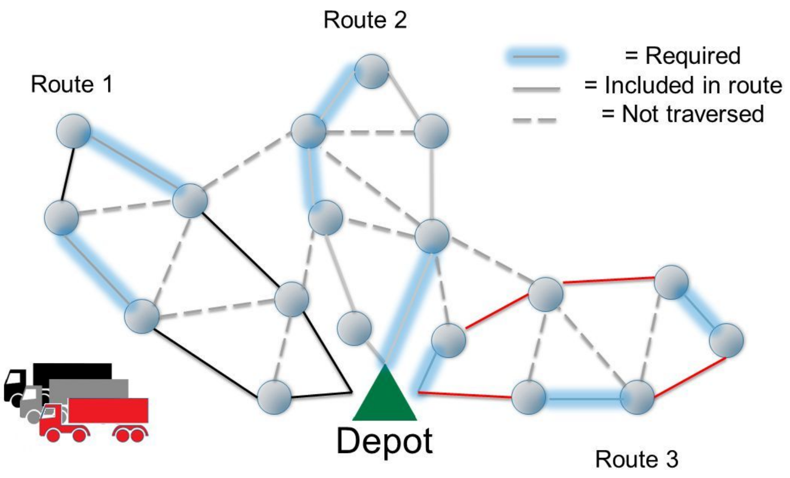
- Generalized Cost-benefit (GCB) algorithm [Zhang and Vorobeychik 2016]
  - Maximizes submodular functions (EER)
  - Imposes *node* routing constraints
  - Runtime approximation guarantee
  - Computationally expensive (faster than brute force)



Example road network

Method (IPP)

- Arc routing
  - Data collection along edges (i.e., continuous sensing)
  - Not a complete graph
  - Required set of edges to visit
  - Might need additional edges to form a path



https://slideplayer.com/slide/7663745/ (figure source)

Method (IPP)

- Arc routing variant
  - Replace TSP with ARP routing solver
  - Can be used in road networks
  - Use deterministic ARP solver (faster convergence)
  - Substantial speedup from Gaussian assumption (EER computation)

Algorithm 1: The modified Generalized Cost-benefit Algorithm (MGCB). $\hat{c}$ is the routing function: TSP when W is the node set V of the graph G, and ARP when W is the edge set E. φ is the submodular cost function (EER), and $W' \backslash x^*$ is the set W' without the element x^*.

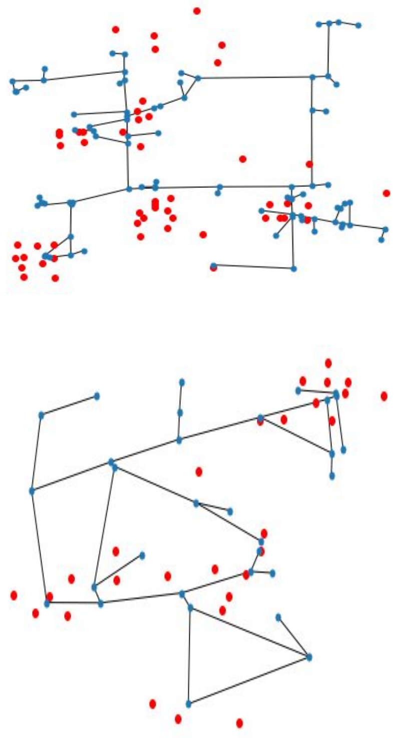
Data: $b > 0, W$
Result: Walk $S \subset W$

$A := \arg\max\{\varphi(x) \mid x \in W, \hat{c}(x) \leq b\}$
$Z := \emptyset$
$W' := W$
while $W' \neq \emptyset$ **do**
    for $x \in W'$ **do**
        $\Delta_\varphi^x := \varphi(Z \cup x) - \varphi(Z)$
        $\Delta_c^x := \hat{c}(Z \cup x) - \hat{c}(Z)$
    end
    $Y := \{x \mid x \in W', \hat{c}\,(Z \cup x) \leq b\}$
    if $|Y| == 0$ **then**
        break
    end
    $x^* = \arg\max\{\Delta_\varphi^x / \Delta_c^x \mid x \in Y\}$
    $Z := Z \cup x^*$
    $W' := W' \backslash x^*$
end
return $\arg\max_{S \in \{A, Z\}} f(S)$

Simulated experimental results

- Dataset
  - Oil well clusters from the Permian basin (U.S.A.)
  - K-means clustering
  - Extracted the road networks
  - 134 total graph

Statistic	Min	Max	Mean
Number of oil wells	11	145	35.50
Number of vertices	15	420	76.28
Number of edges	16	484	81.07
Avg. connectivity [31]	1.0	1.10	1.04



Example graphs

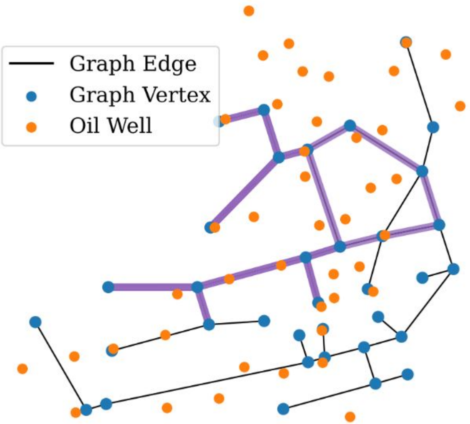
Simulated experimental results

- GCB is faster than path iteration (brute force)
- MGCB at least one order of magnitude faster than GCB
- Converges to the same solution as vanilla GCB

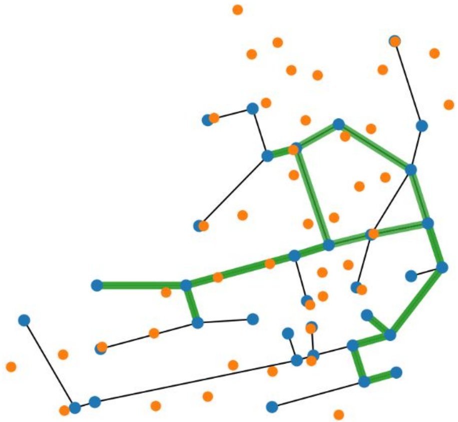
Method	Mean MSE	Std. Dev. of MSE	Mean Time (secs)	Std. Dev. of Time
Path Iter	1.6132E-01	1.2989E+00	1203.81	4.18
GCB TSP	2.0583E-04	1.4948E-03	1209.58	22.64
GCB RPP	2.5680E-05	1.3238E-04	1211.50	17.45
MGCB TSP	2.0583E-04	1.4948E-03	129.41	305.48
MGCB RPP	**2.5680E-05**	1.3238E-04	450.02	512.24

Simulated experimental results

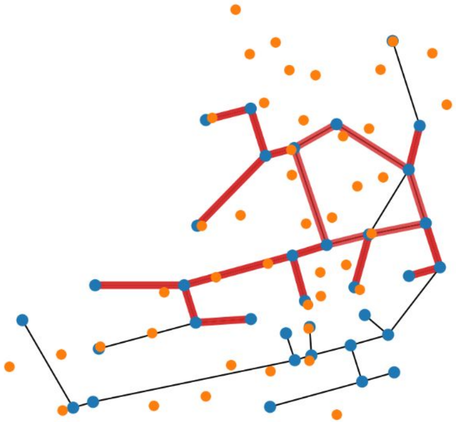
- One of the graphs from our dataset along with the solution from each solver



(a) Path iteration (1.80E-5 MSE)



(b) Modified GCB with TSP (2.20E-5 MSE)



(c) Modified GCB with RPP (1.08E-5 MSE)

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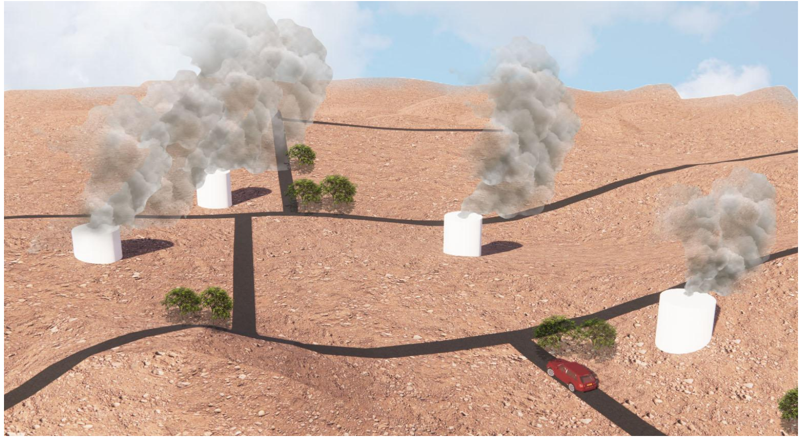
Limitations

- Overlapping gas plumes
  - Leaks from sources in proximity
- Source localization
  - Leaks from pipelines and storage tanks
- Dynamic environmental conditions
  - Changing wind direction, speed, and temperature



Conclusion

- Computationally efficient leak rate estimation
  - Gaussian prior assumption
  - Gas dispersion model factorization
  - Five orders of magnitude faster
- Informative path planning
  - Substantially faster EER computation
  - Leveraged submodularity with the GCB algorithm
  - Generalized GCB to arc routing constraints
  - Improved GCB algorithm’s compute cost
  - At least one magnitude faster than vanilla GCB

Takeaway

- Gaussian assumption
  - Can your phenomena be modeled with a Gaussian?
  - Forest fire smoke dispersion
  - Water pollutants
- Submodularity
  - Is your objective function submodular?
- There are other variants of IPP
  - Does our IPP variant apply to your problem?
- How would you address our limitations
  - Overlapping gas plumes
  - Source localization
  - Dynamic environmental conditions

Acknowledgments and Resources

- Funding: This work was supported in part by NSF Award IIP-1919233

- Paper: https://ieeexplore.ieee.org/document/9706242
- Code: https://github.com/UNCCharlotte-CS-Robotics/Gas-Leak-Estimation

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