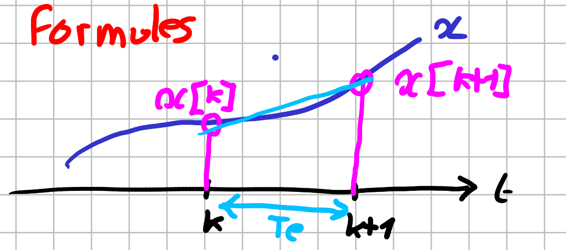
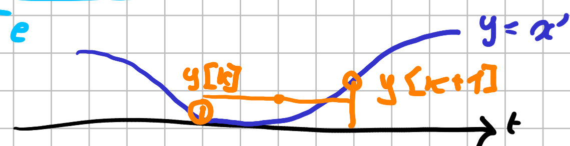
TD SYNTHèse Bilinéaire

1-A Retrouver les Formules



1) pente = $\dfrac{x[k+1] - x[k]}{Te}$



2) moyenne = $\dfrac{y[k+1] + y[k]}{2} = \dfrac{x[k+1] - x[k]}{Te}$

$\mathcal{Z}$

3) $\dfrac{z.Y(z) + Y(z)}{2} = \dfrac{z\,X(z) - X(z)}{Te}$

$Y(z)\,\dfrac{(z+1)}{2} = X(z)\,\dfrac{(z-1)}{Te}$

$\dfrac{Y(z)}{X(z)} = \dfrac{2}{Te}\,\dfrac{z-1}{z+1}$

4) $s(z) = \dfrac{2}{Te}\,\dfrac{z-1}{z+1}$

Equivalents harmoniques

$\boxed{p} \leftrightarrow$ dérivateur $\quad \underbrace{1}_{\text{phaseur}}.e^{i\omega t} \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow \left(e^{i\omega t}\right)' = \underbrace{i\omega}_{\text{phaseur de sortie}}.e^{i\omega t}$

$X(p) \longrightarrow \boxed{p} \longrightarrow Y(p) = p.X(p)$

$e^{i\omega t} \longrightarrow \boxed{j\omega} \longrightarrow j\omega.e^{i\omega t}$ ($p \equiv j\omega$)

donc $p \equiv j\omega$ pour une onde pure

p homogène à $[1/s] = [s^{-1}]$

$\boxed{z} \leftrightarrow$ avance unitaire

$\underbrace{1}_{\text{phaseur}}.e^{i\omega kTe} \longrightarrow \boxed{T^{-1}} \longrightarrow e^{i\omega(k+1)Te} = \underbrace{e^{i\omega Te}}_{\text{phaseur de sortie}}.e^{i\omega kTe}$

$X(z) \longrightarrow \boxed{z} \longrightarrow Y(z) = z.X(z)$

$e^{i\omega kTe} \longrightarrow \boxed{e^{i\omega Te}} \longrightarrow e^{i\omega Te}.e^{i\omega t}$ ($z \equiv e^{i\omega Te}$)

donc $z \equiv e^{j\omega Te}$ pour une onde pure

z sans dimension

Vérifications

1) Homogénéité

$x\,[V] \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow \frac{dx}{dt}\ [V/s]$

$\longrightarrow \boxed{p} \longrightarrow \quad \frac{Y(p)}{X(p)} = p \quad \left[\frac{V/s}{V}\right] = [s^{-1}]$

OK ✓

$$s(z) = \frac{Y(z)\,[V/s]}{X(z)\,[V]} = \frac{2^{[1]}}{T_e\,{}_{[s]}} \; \frac{1+z^{[1]}}{1-z_{[1]}} = [1/s] \quad \text{OK} ✓$$

2) Gain statique

$t \mapsto cte \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow t \mapsto 0 \qquad \underbrace{K}_{\text{gain statique}} = 0$

$$\frac{Y(p)}{X(p)} = p \equiv j\omega \xrightarrow[\underbrace{\omega\to 0}_{\text{statique}}]{} 0 = K \quad \text{OK} ✓$$

$p = 0 \equiv$ statique en continu

$$s(z) = \frac{2}{T_e}\,\frac{1+z}{1-z}$$

$$z \equiv e^{j\omega T_e} \longrightarrow 1 \qquad z = 1 \equiv \text{statique en discret}$$

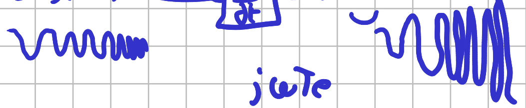
$$s(z) \equiv s\left(e^{j\omega T_e}\right) \xrightarrow[\omega\to 0]{} s(1) = \frac{2}{T_e}\,\frac{1+1}{1-1} = +\infty \quad \text{KO} ✗$$

Et oui ! c'est erreur $\qquad s(z) = \frac{2}{T_e}\,\frac{1-z}{1+z} = \frac{2}{T_e}\,\frac{1-1}{1+1} = 0 = K$ OK ✓

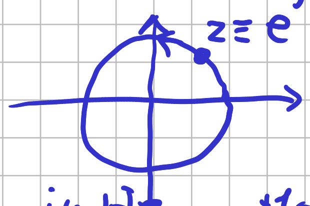
3) Gain HF

$$p \equiv j\omega \xrightarrow[\omega\to+\infty]{} +j\infty \qquad p = +j\infty \text{ pour le gain HF}$$

$\cos(\omega t) \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow -\underset{\smile}{\omega}\,.\sin(\omega t)$ $\qquad K_\infty = +\infty$



$$z \equiv e^{j\omega T_e}$$



$\omega \to +\infty$ n'est pas HF en discret !

$$\omega = 2\pi \underbrace{\frac{F_e}{2}}_{\text{Nyquist}} \longrightarrow e^{i\omega kT_e} \longrightarrow \;{}^{+1}q\;\; p^{+1} \;\; {}_{-1}d \;\; d_{-1}$$

c'est la HF du discret

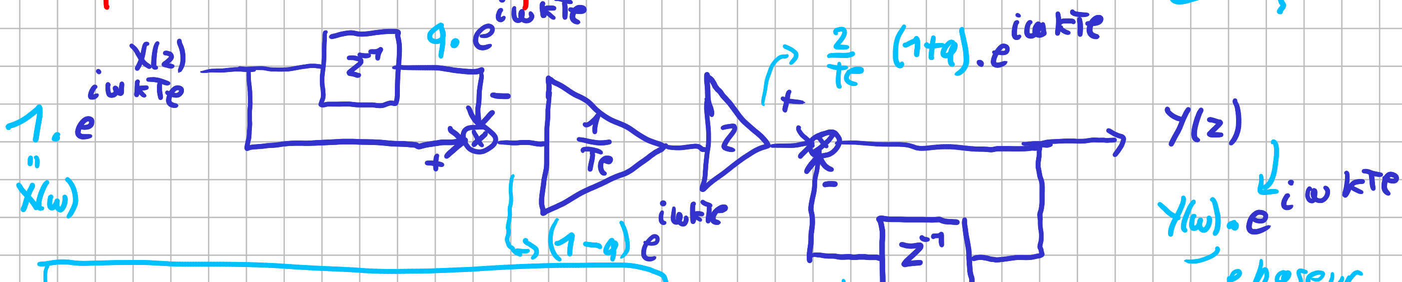
$$z \equiv e^{j2\pi f.T_e} \xrightarrow{f \to \frac{F_e}{2}} e^{j\frac{2\pi}{2}\overbrace{F_e.T_e}^{=1}} = e^{j\pi} = -1$$

En discret on fait $z = -1$ pour avoir le gain HF

$$s(z) \equiv s\left(e^{j\omega T_e}\right) \xrightarrow{\omega \to 2\pi\frac{F_e}{2}} s(-1) = \frac{2}{T_e}\,\frac{\overset{z}{-1} - 1}{\underset{z}{-1} + 1} = \infty \quad \text{OK} ✓$$

Equivalent harmonique de $s(z)$

$$z^{-1} \equiv e^{-i\omega T_e} = q \qquad z = e^{i\omega T_e} = \frac{1}{q}$$



$$\Rightarrow \boxed{1:\; \frac{2}{T_e}(1-q) - q\,Y(\omega) = Y(\omega)}$$

phaseurs

On resoud donc $\frac{2}{Te}(1-q)\cdot X(\omega) = (1+q)\, Y(\omega)$

$$s(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{Te}\,\frac{1+q}{1-q}$$

On ne le ferat plus mais on voit que

$$s(z) = \frac{Y(z)}{X(z)} = \frac{2}{Te}\,\frac{z-1}{z+1} = \frac{2}{Te}\,\frac{1-z^{-1}}{1+z^{-1}} \Rightarrow \frac{Y(\omega)}{X(\omega)} = s(e^{i\omega Te})$$

$$s(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2}{Te}\,\frac{q^{-1}-1}{q^{-1}+1} = \frac{2}{Te}\,\frac{1-q}{1+q} \quad \text{avec } q = e^{-i\omega Te} \equiv z^{-1}, \quad e^{i\omega Te} = \frac{1}{q}$$

Calcul pénible :

$$s(\omega) = \frac{2}{Te}\,\frac{1-e^{-i\omega Te}}{1+e^{-i\omega Te}} = \frac{2}{Te}\,\underbrace{\frac{e^{-i\omega Te/2}}{e^{-i\omega Te/2}}}_{1}\,\frac{e^{+i\omega\frac{Te}{2}} - e^{-i\omega Te/2}}{e^{+i\omega Te/2} + e^{-i\omega Te/2}}$$

$$s(\omega) = \frac{2}{Te}\cdot\frac{2i\sin(\omega Te/2)}{2\cos(\omega Te/2)} = i\,\frac{2}{Te}\tan(\omega Te/2)$$

$$s(z) \equiv s(\omega) = i\,\frac{2}{Te}\tan\left(\omega\frac{Te}{2}\right) \underset{\omega\to 0}{\sim} i\,\frac{2}{Te}\,\omega\,\frac{Te}{2} = i\omega$$

En discret pour $\omega = \omega_d$ $\quad s(z) \equiv i\,\frac{2}{Te}\tan\left(\omega_d\frac{Te}{2}\right) \underset{\omega\to 0}{\sim} i\,\omega_d$

$\wr\wr$

En continu pour $\omega = \omega_c$ $\quad p \equiv j\,\omega_c$

1-B Premier ordre.

0) $G(p) = \frac{k\tau}{1+\tau p}$ on veut $\frac{[V]}{[V]} = [1]$ $\quad p \equiv j\omega$ donc $p\,[s^{-1}]$

on a $\frac{[1][s]}{1+[s][s^{-1}]} = [s]$ PB ! ✗

$\omega = 0 \Rightarrow$ On veut $G(p) \equiv G(j\omega) \xrightarrow[\omega\to 0]{} k$ $\quad G(j\,0) = k\cancel{\tau}$ PB !

On corrige $G(p) = \frac{k}{1+\tau p}$

$\omega = \omega_c \Rightarrow$ idem pour f_c on veut $-3\,dB$

$$\Rightarrow \frac{1}{1+\tau j\omega_c} = \frac{1}{1+j1} \overset{\text{module}}{=} \frac{1}{\sqrt{2}} \qquad \text{On corrige } \tau = \frac{1}{\omega_c} = \frac{1}{2\pi f_c}$$

$$G(p) = \frac{k}{1+\tau p}, \quad 2\pi\tau = \frac{1}{f_c}$$

1) $$G_s(z) = G\left(\frac{2}{Te}\,\frac{z-1}{z+1}\right) = \frac{k}{1+2\frac{\tau}{Te}\,\frac{z-1}{z+1}} = k\,\frac{(1+z)}{1+z+2\frac{\tau}{Te}(z-1)}$$

— issu de la correction

$$G_s(z) = k \, \frac{\overset{[1]}{1} + \overset{[1]}{z}}{\underset{[1]}{1} - \underset{[1]}{2\frac{\tau}{T_e}} + \underset{[1]}{\left(1+2\frac{\tau}{T_e}\right)} \underset{[1]}{z}}$$

$G_s(1) = k \dfrac{2}{2} = k$ ✓

$G_s(-1) = k \dfrac{0}{-2\tau/T_e} = 0$ ✓

$G_s(z)$ [1] homogène ✓

$$G_s(z) = \underset{[1]}{\frac{k}{1-2\frac{\tau}{T_e}}} \cdot \frac{1+z}{1 + \underset{[1]}{\left(\frac{T_e+2\tau}{T_e-2\tau}\right)} z}$$

$G_s(1) = \dfrac{k\,2}{1-2\frac{\tau}{T_e} + T_e+2\tau} = k$ ouf !

$G_s(-1) = 0$ ✓

$G_s(z)$ [1] ✓

⚠

$$G_s(1/z^{-1}) = \overset{[s]}{\frac{k.T_e}{T_e+2\tau}}\,[s] \;\; \frac{1+z^{-1}}{1 + \underset{[1]}{\left(\frac{T_e-2\tau}{T_e+2\tau}\right)} z^{-1}}$$

$G_s(1) = \dfrac{2k}{2} = k$ ✓

$G_s(-1) = k\dfrac{0}{4\tau} = 0$ ✓

$G_s(z)$ [1] ✓

$$G_s(1/q) = k' \, \frac{1+q}{1+a_1 q}$$

avec $k' = k.\dfrac{T_e}{T_e+2\tau}$ [1] $\neq$ gain stat. en discret

$a_1 = \dfrac{T_e-2\tau}{T_e+2\tau}$ [1]

5) $Y(z)\left(1 + a_1 z^{-1}\right) = k' X(z)\left(1+z^{-1}\right)$

$z^{-1} \downarrow$

$$y[k] + a_1 \, y[k-1] = k'\left(x[k] + x[k-1]\right)$$

$$\overset{[V]}{y[k]} = \underbrace{-\overset{[1]}{a_1}\,\overset{[V]}{y[k-1]}}_{\text{Auto Regressive}} + \overset{[1]}{k'} \underbrace{\left(\overset{[V]}{x[k]} + \overset{[V]}{x[k-1]}\right)}_{\text{Mooving Average}}$$

homogène

6) Théorème suite extraite « si $(U_n)_{n\in\mathbb{N}} \to \ell$ » donc si stable ici

$(y_n)_{n\in\mathbb{N}} \to y^*$ et $(y_{n+1}) \to y^*$ etc. en statique $(x_n = x^*)$

$$y^* = -a_1 y^* + k'(x^* + x^*) \Rightarrow y^* = \frac{2k'}{1+a_1} x^*$$

$$y^* = \frac{2.\frac{k T_e}{T_e+2\tau}}{1 + \frac{T_e-2\tau}{T_e+2\tau}} x^* = \frac{2kT_e}{T_e+2\tau+T_e-2\tau} = 2k\frac{T_e}{2T_e} = 2\frac{k}{2} = k$$ ✓ gain statique

7) On sait qu'aux HF $\overrightarrow{\omega_{Fe/2}} = (1,-1,1,-1) \Rightarrow x[k] = -x[k-1] = \dots = x^*$

en entrée

$$x^*.\overrightarrow{\omega_{Fe/2}} \longrightarrow \boxed{H} \longrightarrow \underbrace{x^*.H(\omega_{Fe/2})}_{y^*}.\overrightarrow{\omega_{Fe/2}}$$

En sortie on a aussi du $y^*.\overrightarrow{\omega_{Fe/2}} \Rightarrow y[k] = -y[k-1] = \dots = y^*$

$(y_{2n}) \longrightarrow y^*$ et $(y_{2n+1}) \longrightarrow -y^*$ si stable !

$$y[k+1] = a_1 \; y[k] - k'(x[k+1] + x[k])$$

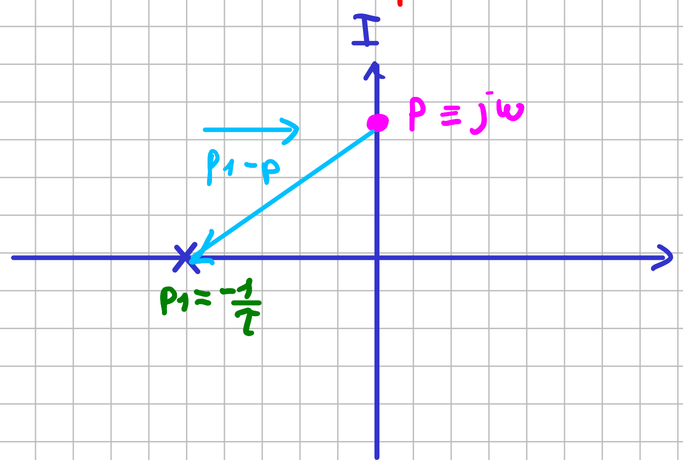
$\overrightarrow{\omega_{\frac{f_e}{2}}}$

$$y^* = a_1 \cdot -y^* - k'(x^* + -x^*)$$

$$y^* = -\frac{k'}{1+a_1} \cdot 0 = 0$$

✓ gain HF du passe-bas

Réponse harmonique



$$G(p) = \frac{K}{1+\tau_p} = K \cdot \frac{1}{1-\frac{p}{p_1}} = K \frac{p_1}{p-p_1}$$

$p_1 = -\frac{1}{\tau}$ pôle ×

$p \equiv j\omega$

$$\angle G(p) = \underset{\underset{0 \text{ ou } \pi}{=}}{\angle k} + \underset{\underset{\pi}{=}}{\angle p_1} - \angle \overrightarrow{p_1 - p}$$

$$|G(p)| = |K| \cdot \left|\frac{p_1}{p_1 - p}\right|$$

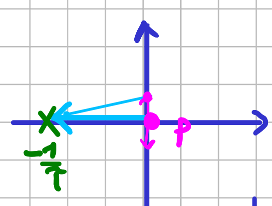
dB

$$|G(p)|_{dB} = |K|_{dB} + |p_1|_{dB} - |\overrightarrow{p_1 - p}|_{dB}$$

Asymptotique

$p \equiv j\omega \xrightarrow{\omega \to 0} j0$

statique



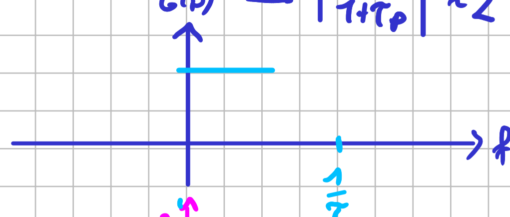
$|p_1 - p| \approx |p_1| \Rightarrow$ pôle$_{dB} = |p_1|_{dB} - |p_1|_{dB} = 0\,dB$

$$\left|\frac{1}{1+\tau_p}\right| \approx \frac{1}{1+\tau j\omega} \underset{\text{si } \omega\tau \ll 1}{\approx} 1$$

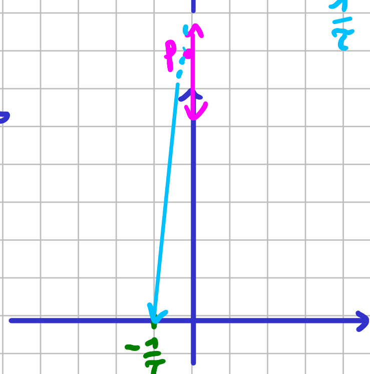
$\angle p_1 - p = \pi \Rightarrow \angle$ pôle $= \angle p_1 - \angle p_1 - p = \pi - \pi = 0$

$$G(p) \; \angle \left|\frac{1}{1+\tau_p}\right| \approx \angle \frac{1}{1+\tau j\omega} \underset{\omega \ll \frac{1}{\tau}}{\approx} \angle 1 = 0$$

$$\frac{k}{1+\tau_p} \underset{\omega \ll \tau}{\approx} k$$



HF $p \equiv j\omega \xrightarrow{\omega \to +\infty} +j\infty$



$\angle p_1 - p = -\frac{\pi}{2} \Rightarrow \angle$ pôle $= \pi - \frac{\pi}{2} = \frac{\pi}{2}$

$|p_1 - p| \approx |p| = |j\omega| = \omega$

$\frac{1}{\tau} \ll j\omega$

$\Rightarrow |$pôle$| \propto \frac{1}{\omega}$

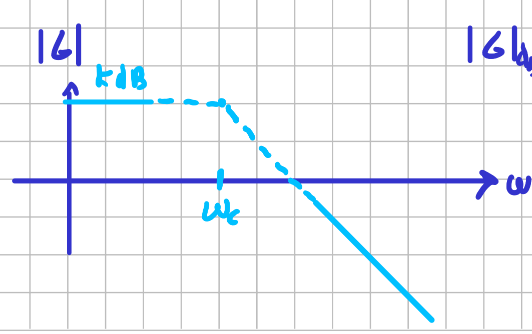
$|$pôle$|_{dB} = |p_1|_{dB} + \left|\frac{1}{\omega}\right|_{dB} = -20\,dB/dec$

$\omega \times 10 \Rightarrow \frac{|\text{pôle}|}{10}$

$$\frac{k}{1+T_p} = \frac{k}{1+\tau j\omega} \underset{1 \ll \tau\omega}{\approx} \frac{k}{\tau j\omega} = -j\frac{k}{\tau\omega} = k \cdot \left(-j\frac{\omega_c}{\omega}\right)$$

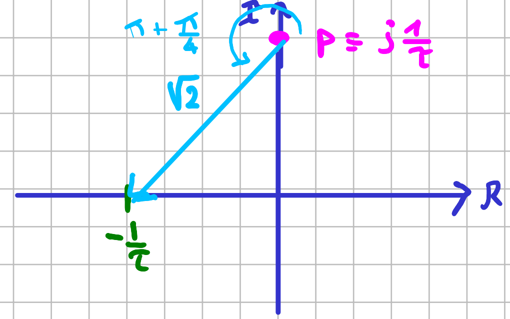
$\angle G = -\pi/2$

$|G|_{dB} = K_{dB} - \left|\frac{\omega}{\omega_c}\right|_{dB}$



Cassure

$\omega = \omega_c$



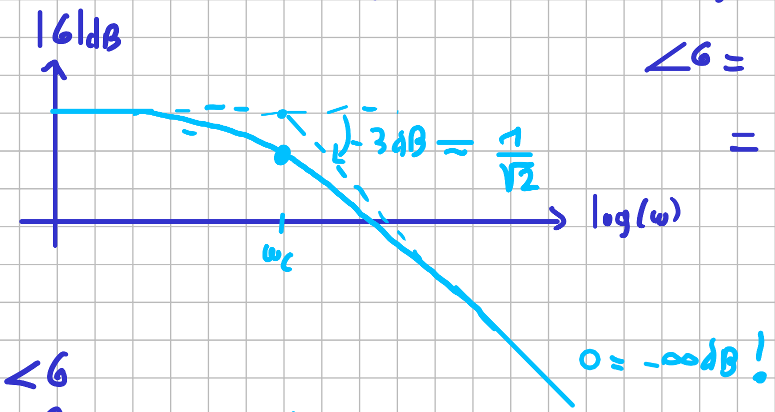
$|\vec{p\hat{o}le}| = \sqrt{2}$

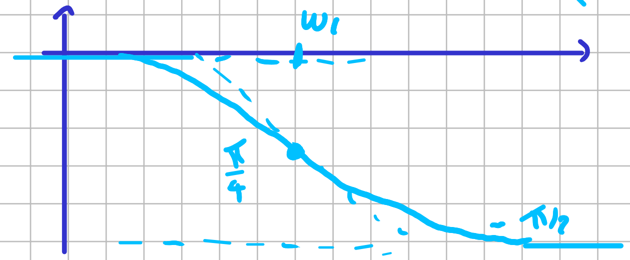
$\angle \vec{p\hat{o}le} = \pi - \left(\pi + \frac{\pi}{4}\right) = -\frac{\pi}{4}$

$\omega = \omega_c \Rightarrow G(j\omega) = \dfrac{k}{1 + j\omega.\tau.\omega_c} = \dfrac{k}{1+j}$ $\qquad \omega_c = \frac{1}{\tau}$

$\angle G = \angle k - \angle 1+j$

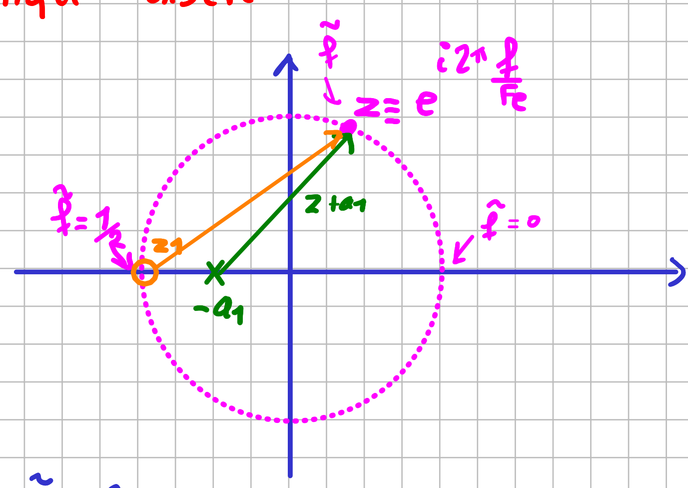
$= 0 - \frac{\pi}{4}$





Reponse harmonique discret

$z \equiv e^{jTe\omega} = e^{j2\pi\tilde{f}}$



$G_s(1/q) = k' \dfrac{1+q}{1+a_1 q}$

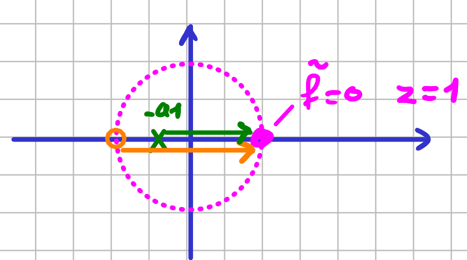
$= k' \dfrac{z+1}{z+a_1}$

$p_1 = -a_1$

$z_1 = -1$

$= k'.-1 \dfrac{z-(-1)}{z-(-a_1)}$

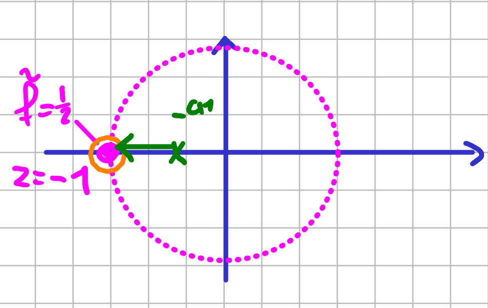
Statique $\quad z \equiv e^{-j2\pi\tilde{f}} \xrightarrow{\tilde{f} \to 0} 1 \qquad G_s(z) \equiv k' \dfrac{1+1}{1+a_1}$



$|G_s| = |k'| \dfrac{2}{a_1+1} = \dfrac{2k'}{a_1+1} = K$ ✓

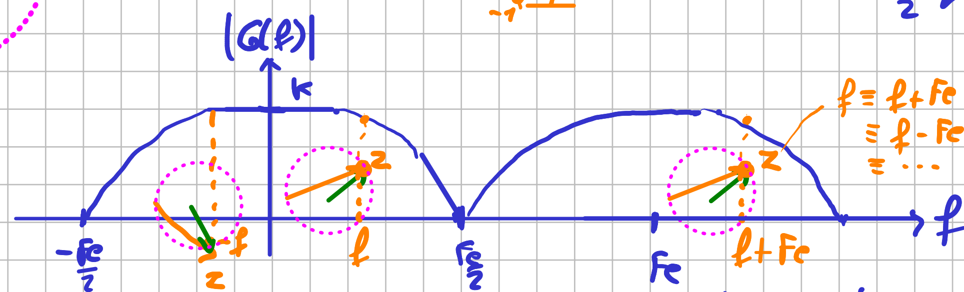
$\angle G_s = 0° - 0° = 0°$ ✓

HF



$|G_s| = |k| . \dfrac{0}{a_1} = 0$ ✓

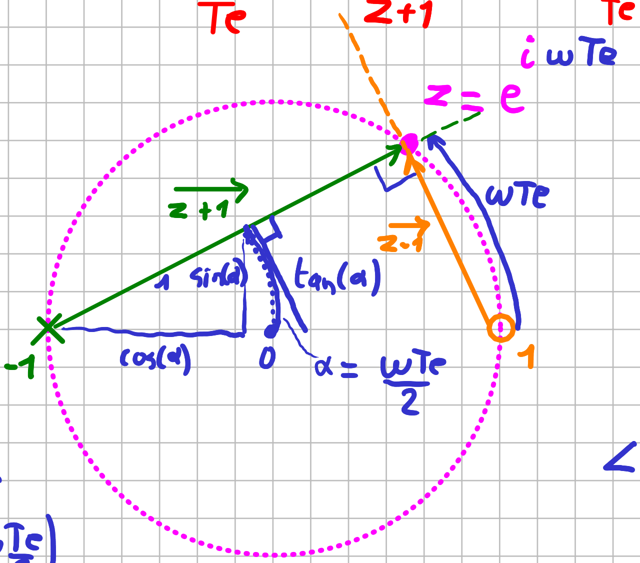
$\angle G_s = \pi/2 - \pi = -\frac{\pi}{2}$ ✓



$z \equiv e^{i2\pi\tilde{f}} = \overline{e^{i2\pi(-\tilde{f})}} \Longrightarrow G(-f) = \overline{G(f)} \qquad G(f+Fe) = G(f)$

Retour sur $s(z) = \frac{2}{Te} \frac{z-1}{z+1} = \frac{2}{Te} \frac{z-(1)}{z-(-1)}$ zero $= 1$ o

$\hat{p}_1 = -1$ x



$|s(z)| = \frac{2}{Te} \frac{|z-1|}{|z+1|} = \tan\alpha$

$= \tan\left(\omega \frac{Te}{2}\right)$

$\angle s(z) \equiv \angle z-1 - \angle z+1$

$\angle z-1 = \frac{\pi}{2} + \alpha$, $\angle z+1 = \alpha$

$\angle s(z) \equiv \frac{\pi}{2} \Rightarrow s(z) \equiv i \cdots^{>0}$

$$s(z) \equiv j \frac{2}{Te} \tan\left(\omega \frac{Te}{2}\right)$$

Pour ω_c $\quad s(z) \equiv j \underbrace{\frac{2}{Te} \tan\left(\omega_c \frac{Te}{2}\right)}_{\omega_d} = j\,\omega_d$ "pour discret"

$\neq j\,\omega_c$

Donc $G(s(z)) = G(j\omega_d) \neq G(j\omega_d) = \frac{K}{1+i}$

Prewarp : On prend $G_{pre} = \frac{K}{1+\tau_{pre}\,p}$ tel que $G_{pre}(j\omega_d) \underset{=}{=} \frac{K}{1+i}$

Ainsi $G_{pre}(s(z)) \underset{\omega \to \omega_c}{\equiv} G_{pre}\Big(j\omega_d = j \underbrace{\frac{2}{Te} \tan\left(\omega_c \frac{Te}{2}\right)}_{\omega d}\Big) = \frac{K}{1+i}$

Donc $\tau_{pre} = \frac{1}{\omega_d} = \frac{1}{\frac{2}{Te}\tan\left(\omega_c \frac{Te}{2}\right)}$

si $Fe \nearrow$, $Te \searrow$ $\quad \omega_c \frac{Te}{2} \searrow$ et $\ll 1 \Rightarrow \omega d \underset{\omega_c}{\sim} \omega_c$

prewarp inutile

sinon prewarp

$$\boxed{\begin{array}{ll} G(z) = G_{pre}(s(z)) = K' \dfrac{1+z^{-1}}{1+a_1 z^{-1}} & K' = K \dfrac{Te}{Te + 2\tau_{pre}} \\ \tau_{pre} = \dfrac{1}{\omega_d} \text{ car } \omega_d = \dfrac{2}{Te}\tan\left(\omega_c \dfrac{Te}{2}\right) & a_1 = \dfrac{Te - 2\tau_{pre}}{Te + 2\tau_{pre}} \\ G(\omega \to 0) = K, \quad G\left(\omega \to \dfrac{\omega_e}{2}\right) = 0, \quad G(\omega \to \omega_c) = \dfrac{K}{1+i} & \end{array}}$$