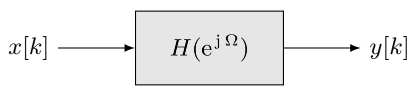
Ensemble Averages	
First Order	$E\{f(x[k])\} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} f(x_n[k])$
Linear Mean	$\mu_x[k] = E\{x[k]\}$
Quadratic Mean	$E\{x^2[k]\}$
Variance	$\sigma_x^2[k] = E\{(x[k] - \mu_x[k])^2\} = E\{x^2[k]\} - \mu_x^2[k]$
Second Order Order	$E\{f(x[k_1], x[k_2])\} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} f(x_n[k_1], x_n[k_2])$
Auto-Correlation Function (ACF)	$\varphi_{xx}[k_1, k_2] = E\{x[k_1] \cdot x[k_2]\}$
Properties	
Linearity	$E\{a \cdot x[k] + b \cdot y[k]\} = a \cdot E\{x[k]\} + b \cdot E\{y[k]\}$
Deterministic Signal $s[k]$	$E\{s[k]\} = s[k]$

Stationary and Ergodic Processes	
Stationarity	$E\{f(x[k_1], x[k_2])\} = E\{f(x[k_1 + \Delta], x[k_2 + \Delta])\}$
First Order Ensemble Average	$E\{f(x[k_1])\} = E\{f(x[k_1 + \Delta])\}$
Linear Mean	$\mu_x[k] = \mu_x$
Variance	$\sigma_x^2[k] = \sigma_x^2$
Auto-Correlation Function (ACF)	$\varphi_{xx}[\kappa] = E\{x[k] \cdot x[k - \kappa]\} = E\{x[k + \kappa] \cdot x[k]\}$
Cross-Correlation Function (CCF)	$\varphi_{xy}[\kappa] = E\{x[k + \kappa] \cdot y[k]\} = E\{x[k] \cdot y[k - \kappa]\}$
Power Spectral Density (PSD)	$\Phi_{xx}(e^{j\Omega}) = \mathcal{F}_*\{\varphi_{xx}[\kappa]\}$
Cross Power Spectral Density (CSD)	$\Phi_{xy}(e^{j\Omega}) = \mathcal{F}_*\{\varphi_{xy}[\kappa]\}$
Ergodicity	$\overline{f(x_n[k], x_n[k - \kappa_1], x_n[k - \kappa_2], \dots)}$
	$= E\{f(x[k], x[k - \kappa_1], x[k - \kappa_2], \dots)\} \quad \forall n$

Random Signals and LTI Systems



Stationary Input $x[k]$

Linear Mean $\quad \mu_y = \mu_x \cdot H(e^{j \cdot 0})$

Correlation Functions and Power Spectral Densities

$\varphi_{xx}[\kappa]$	○—●	$\Phi_{xx}(e^{j\Omega})$
$\varphi_{yx}[\kappa] = h[\kappa] * \varphi_{xx}[\kappa]$	○—●	$\Phi_{yx}(e^{j\Omega}) = H(e^{j\Omega}) \cdot \Phi_{xx}(e^{j\Omega})$
$\varphi_{xy}[\kappa] = h^*[-\kappa] * \varphi_{xx}[\kappa]$	○—●	$\Phi_{xy}(e^{j\Omega}) = H^*(e^{j\Omega}) \cdot \Phi_{xx}(e^{j\Omega})$
$\varphi_{yy}[\kappa] = \varphi_{hh}[\kappa] * \varphi_{xx}[\kappa]$	○—●	$\Phi_{yy}(e^{j\Omega}) = \lvert H(e^{j\Omega})\rvert^2 \cdot \Phi_{xx}(e^{j\Omega})$

where $\varphi_{hh}[\kappa] = h[\kappa] * h^*[-\kappa]$

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Amplitude Distribution	
Cumulative Distribution Function (CDF)	
Univariate	$P_x(\theta, k) = \mathcal{W}\{x[k] \leq \theta\}$
Stationary Process	$P_x(\theta, k) = P_x(\theta)$
Bivariate	$P_{x_1 x_2}(\theta_1, \theta_2, k_1, k_2) = \mathcal{W}\{(x_1[k_1] \leq \theta_1) \wedge (x_2[k_2] \leq \theta_2)\}$
Stationary Process	$P_{x_1 x_2}(\theta_1, \theta_2, k_1, k_2) = P_{x_1 x_2}(\theta_1, \theta_2, \kappa)$ mit $\kappa = k_2 - k_1$
Probability Density Function (PDF)	
Univariate	$p_x(\theta, k) = \frac{\mathrm{d}}{\mathrm{d}\theta} P_x(\theta, k)$
Bivariate	$p_x(\theta_1, \theta_2, k_1, k_2) = \frac{\partial^2}{\partial\theta_1 \partial\theta_2} P_{x_1 x_2}(\theta_1, \theta_2, k_1, k_2)$
Ensemble Averages for a Stationary Process	
First Order	$E\{f(x[k])\} = \int_{-\infty}^{\infty} f(\theta)\; p_x(\theta)\, \mathrm{d}\theta$
Second Order	$E\{f(x_1[k], x_2[k+\kappa])\} = \iint_{-\infty}^{\infty} f(\theta_1, \theta_2)\; p_{x_1 x_2}(\theta_1, \theta_2, \kappa)\, \mathrm{d}\theta_1\, \mathrm{d}\theta_2$
Linear Mean	$\mu_x = E\{x[k]\} = \int_{-\infty}^{\infty} \theta\; p_x(\theta)\, \mathrm{d}\theta$
Variance	$\sigma_x^2 = E\{x^2[k]\} - \mu_x^2 = \int_{-\infty}^{\infty} \theta^2\; p_x(\theta)\, \mathrm{d}\theta - \mu_x^2$
Auto-Correlation Function	$\varphi_{xx}[\kappa] = E\{x[k] \cdot x[k-\kappa]\} = \iint_{-\infty}^{\infty} \theta_1 \theta_2\; p_x(\theta_1, \theta_2, \kappa)\, \mathrm{d}\theta_1\, \mathrm{d}\theta_2$

Selected Amplitude Distributions	
Uniform Distribution	$p_x(\theta) = \begin{cases} \frac{1}{x_o - x_u} & \text{for } x_u < \theta \leq x_o \\ 0 & \text{otherwise} \end{cases}$ $P_x(\theta) = \begin{cases} 0 & \text{for } \theta \leq x_u \\ \frac{\theta - x_u}{x_o - x_u} & \text{for } x_u < \theta \leq x_o \\ 1 & \text{for } \theta > x_o \end{cases}$ $\mu_x = \frac{x_o - x_u}{2},\ \sigma_x^2 = \frac{(x_o - x_u)^2}{12}$
Normal (Gaussian) Distribution	$p_x(\theta) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(\theta - \mu_x)^2}{2\sigma_x^2}}$ $P_x(\theta) = \frac{1}{\sqrt{2\pi}\sigma_x} \int_{-\infty}^{\theta} e^{-\frac{(\xi - \mu_x)^2}{2\sigma_x^2}}\, \mathrm{d}\xi$
Laplace Distribution	$p_x(\theta) = \frac{1}{\sqrt{2}\sigma_x} e^{-\sqrt{2}\frac{\lvert\theta - \mu_x\rvert}{\sigma_x}}$ $P_x(\theta) = \begin{cases} \frac{1}{2} e^{\sqrt{2}\frac{\theta - \mu_x}{\sigma_x}} & \text{for } \theta \leq \mu_x \\ 1 - \frac{1}{2} e^{-\sqrt{2}\frac{\theta - \mu_x}{\sigma_x}} & \text{for } \theta > \mu_x \end{cases}$

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