Optimal Liquidity Provision and Interest Rate Rules

Preliminary and not to be quoted without permission

Paul Levine[1], Maryam Mirfatah [2], Joseph Pearlman[3], and Stylianos Tsiaras[4]

[1]University of Surrey
[2]King's College London
[3]City University London
[4]Ecole Polytechnique Fédérale de Lausanne

January 25, 2022

Abstract

This paper analyses and estimates optimized simple and implementable liquidity and interest rates rules that maximize welfare. We employ a DSGE model, estimated for the Euro Area, with financial frictions on the supply and demand side of credit where liquidity provision could be welfare reducing due to the existence of the risk-taking channel. We show that our estimated Taylor-type liquidity rule linked to output, inflation and spreads increases welfare, eliminates the contractionary effects and stimulates the macroeconomy in contrast to a simple liquidity rule. Furthermore, we estimate an optimized monetary rule that is also linked to spreads. Our findings suggest that introducing liquidity provision policy alongside the standard monetary rule is welfare enhancing.

Contents

1 Introduction

Since the onset of the Great Recession, and also during the times of the recent COVID-19 pandemic, central banks in the US and the Euro area have employed a number of non-standard monetary policy tools. The extension of existing reverse operations under longer maturities and the asset purchase programs were the most popular among those tools. In this paper we focus on the liquidity provisions, a scheme intensively used by the European Central Bank; in the ECB's jargon: the Long Term Refinancing Operations (LTROs). Although the key scope of these direct funding programs was the stabilization of economic activity through a credit expansion, the existence of the risk-taking channel could make liquidity provision in turbulent times contractionary and reduce welfare.[1] We explore a simple and implementable liquidity rule that responds to output, inflation and credit spread changes. We estimate a model with financial frictions on the supply and demand side of credit with Euro Area data and find that our liquidity rule maximizes welfare, eliminates the potential contractionary effects of the liquidity injections and is stimulative for the macroeconomy.

This study introduces agency problems associated with financial intermediation in an otherwise standard business cycles model and estimates the model for the Euro Area. A modelling framework is presented where banks are able to receive emergency liquidity funds from the central bank. By combining Gertler and Kiyotaki (2010) with Bernanke, Gertler, and Gilchrist (1999) (henceforth GK and BGG respectively) a setting is developed where the financial frictions on the supply and demand side of credit have different impact on the macroeconomy. Our framework identifies the two opposing forces after a liquidity injection. We frame liquidity injections similarly to the liquidity facilities introduced by GK. In our model, liquidity loosens the GK banks' friction and stimulates the macroeconomy after a negative shock. Concurrently, due to the BGG friction more liquidity and thus more credit to the real economy leads to more loans to be repaid by the firms. Due to the contractionary effects of the shock, some firms have low net worth and are unable to pay back their loans. This increases the firms' default rate and leads to welfare contraction. When the latter effect originating from the BGG friction is stronger than the loosening of the financial constraint, we could have a contraction after a liquidity injection.

We introduce and analyse a liquidity and a monetary rule that are welfare maximizing and bypass the potentially negative effects of the BGG friction. Our framework assumes that both rules can respond to deviations in output, inflation and the lending spread from their steady state equilibrium. Our specification then is similar to a conventional Taylor monetary rule assumed in New Keynesian models. In our estimated model we find the liquidity parameter weights together with the interest rate rule parameters that maximize welfare. To provide policy advice, we decompose the optimal liquidity rule effects into

[1] See Acharya, Eisert, Eufinger, and Hirsch (2019) and Tsiaras (2018).

its different response variables. When liquidity responds only to some and not all the variables included in the rule, the rule is not welfare maximizing any more.

To identify the main result of our paper, namely a liquidity rule that maximizes welfare, we proceed with the following methodological approach: firstly we estimate our model in absence of the liquidity rule which is our baseline specification. We then set our model's parameters to the estimated ones and add the liquidity rule and search for the rule parameter values that maximize welfare. Lastly we stimulate and compare both models for a large time period and compare them in terms of welfare. We find that for certain rule parameter values liquidity rule is welfare maximizing.

As described above, there are two frictions into play in our model: liquidity relaxes the friction between households and banks and builds up the one between firms and banks. We show that our rule is welfare enhancing conditional to a high monitoring rate of the liquidity funds. This occurs when the bank cannot abscond a high proportion of the liquidity provided and thus liquidity relaxes the banking friction substantially.

To give a much clearer policy advice and to identify which rule framework dominates from a welfare perspective we proceed our analysis by decomposing the liquidity rule to its three components. At every iteration of the exercise we allow only one component to be active while we assume that the central bank does not respond to the remaining components. We find that the rule that responds to all three variables namely output, inflation and credit spread is the one that achieves a higher welfare compared to the other specifications.

Finally, in order to gain some detailed intuition of how liquidity affects our economy in a non-stochastic environment we perform a steady state analysis of the economy. As liquidity increases from zero to its limit, it is beneficial for the macroeconomy and is loosening the lending spread thus providing more credit to the non financial firms. At the same time it increases the probability of default of the firms due to the higher credit they receive. This result verifies the two opposite forces of our two financial frictions specification, and provides motivation for the search of an optimized rule that will be welfare enhancing. We also study the effects of a negative liquidity injection. This is simply a lending from the banks to the central bank similarly to the reserves (or the deposit facility in the ECB framework). This facility also benefits the main macro variables but at a less magnitude than the liquidity facility.

As a side result of our model, and to contribute to the recent monetary - macro-prudential policy cooperation discussion we study a classical monetary rule that is augmented with a response to lending spread deviations additional to output and inflation. Thus adding a macro-prudential response to the classical policy rate specification.[2] We find that when the policy rate responds to spread deviations the economy recovers faster

[2] Laureys and Meeks (2018) do a similar exercise where the policy rate responds to the loan-to-output rate as another way of introducing macroprudential measure in the policy rate rule.

after a negative shock.

In the last part of the paper we provide a brief analysis of the conditions that guarantee the uniqueness of equilibrium. We do that for the case of the liquidity rule and also the policy rate rule. We show how the liquidity rule but also the presence of spread deviations in the policy rate rule can alter the equilibrium properties of our financial frictions model. In particular, under certain parameter configurations for the spread deviation the economys equilibrium may be indeterminate even when the interest rate rule is one that satisfies the Taylor principle.

Related Literature. Macroeconomic models with financial frictions have populated a substantial fraction of the macro literature after the Great Recession following the seminal papers of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) (see Gertler and Kiyotaki (2010), Sims and Wu (2021) among many others).[3] Prior to the financial frictions models, most of the existing modern macroeconomic models did not take into account financial frictions between households, firms and the banking sector. In this paper, we analyse a model with financial frictions on the supply and the demand side of credit by combining the seminal models of Bernanke et al. (1999) and Gertler and Kiyotaki (2010). A model relatively close to ours without the unconventional monetary policy component is Rannenberg (2016). He shows that the model matches the data relatively better and outperforms both a BGG and a GK-type model. We identify two opposing forces arising from the two frictions and the risk-shifting channel when liquidity is provided to the banks and find a liquidity rule that is welfare enhancing and maximizing.

While studies on optimal setting of monetary policy populated the literature in the past, studies on optimal macroprudential rules are a recent strand of the literature. Ferrero, Harrison, and Nelson (2018b) study the optimal setting of a loan to value ratio in a New Keynesian model with price rigidities and financial frictions and Ferrero, Harrison, and Nelson (2018a) study the optimal interaction of monetary and macroprudential policy. Angelini, Neri, and Panetta (2014) study the interaction of capital requirements and monetary policy. Rather than macroprudential policy, our focus on optimal liquidity and its interaction with interest rate rules in an estimated New Keynesian model with financial frictions.

The risk-shifting channel of monetary policy, has regained attention after the Great Recession which has been characterised from substantial monetary easing from the central banks. Allen and Gale (2000), Diamond and Rajan (2012) were among the first to identify the risk-shifting channel of monetary policy. In an empirical framework Jiménez, Ongena, Peydró, and Saurina (2014) and Ioannidou, Ongena, and Peydró (2014) find that monetary expansion induces banks to grant loans to more risky firms which increases the likelihood of

[3] Also Eggertsson and Woodford (2003), Curdia and Woodford (2011), Gertler and Karadi (2011). For a comprehensive literature review on the developments of models with financial factors see Gertler and Gilchrist (2018).

default. Dell'Ariccia, Laeven, and Suarez (2017) find similar results for the U.S. [4] Adrian and Shin (2010) build a theoretical model and show that expansionary monetary policy increases the risk taking of the banking sector by relaxing the bank capital constraint due to moral hazard problems.

Finally, there are studies on the ECB's LTRO which are close to the subject of this paper. Cahn, Matheron, and Sahuc (2017), Joyce, Miles, Scott, and Vayanos (2012), Bocola (2016), van der Kwaak (2017) to name a few. They include financial frictions following the Gertler and Karadi (2011) seminal paper. In such a setting liquidity is always beneficial because it relaxes the banks' constraint. Our framework which includes also a friction on the demand for credit identifies a potential contractionary effect of the LTROs. We propose a liquidity rule that responds not only to credit spread as in the aforementioned papers but also to inflation and output, eliminates any contractionary effects and is welfare maximizing.

The outline of the paper is as follows. Section 2 describes the financial frictions component of the model together with the liquidity rule framework. Section 3 presents the estimation results of the model, the data used and the measures of fit. The following sections present the main results. We first perform a steady state analysis in Section 4 to understand the model's behaviour for all the possible values of the liquidity provision. In Section we present various quantitative exercises for the model without the liquidity rule active and see how it responds to shocks. Sections 5 and 6 describle the general framework of welfare-optimized simple rules and the delegation game that imposes a zero-lower-bound on the nominal interest rate in an equilibrium. Section 7 outlines the welfare optimizing liquidity rule and performs a determinacy analysis. We then break the rule down to its components to study which rule specification performs relatively better. The last section concludes.

2 The Financial Frictions Model

The model combines the the banks-firms asymmetric information framework of BGG and the banks-households limited commitment problem of GK. This setting is incorporated into an New Keynesian model with monopolistic competition, sticky prices and sticky wages similar to Smets and Wouters (2007). In this Section we describe only the financial frictions component of the model. Appendix A outlines the NK part of the model.

The financial frictions economy is populated by a continuum of financial intermediaries owned by households. There is a continuum of entrepreneurs that own the non-financial firms. A monetary authority and the treasury complete this part of the economy. There

[4] For more studies that identify the risk-taking channels see: Delis, Hasan, and Mylonidis (2017), Buch, Eickmeier, and Prieto (2014), Altunbas, Gambacorta, and Marques-Ibanez (2010), Maddaloni and Peydró (2011) and Lown and Morgan (2006) among others.

is a moral hazard problem between the households and the banks. Banks can steal a fraction of their funds and return them to their families. This problem introduces an incentive constraint to the model to be followed by the banks. The second financial friction originates from a firm-bank problem. Entrepreneurs at every period receive an idiosyncratic shock that change the value of their assets. Low values of the shock can lead to default on their credit. Finally, the central bank performs its conventional monetary policy under a Taylor rule, but can also provide liquidity following our liquidity rule.

2.1 The Debt-Contracting Problem

At every period period there is a fixed mass of intermediaries indexed by e. Each period, a fraction $1-\sigma_E$ of entrepreneurs, exit and give retain earnings to their household. An equal number of new entrepreneurs enter at the same time. They begin with a start up fund of ξ_E given to them by their household. The entrepreneur e (the non-financial firm) seeks loans $L_{e,t}$ to bridge the gap between its net worth $N_{E,e,t}$ and the expenditure on new capital $Q_t K_{e,t}$, all end-of-period. Thus

$$L_{e,t} = Q_t k_{e,t} - N_{E,e,t} \tag{1}$$

where the entrepreneur's *real* net worth accumulates according to

$$N_{E,e,t} = R_t^K Q_{t-1} K_{e,t-1} - \frac{R_{t-1}^L}{\Pi_t} L_{e,t-1}$$

where R_t^K is the gross real return on capital as in the NK model and R_t^L is the *nominal* loan rate to be decided in the contract. Each entrepreneur determines the utilization rate u_t and provides an effective amount of capital to the firms for production, getting bank the rental rate of capital r_t^K. At the end of the production schedule, the capital is being resold to capital goods producers at price Q_t. Then the gross real return on capital is defined as:

$$R_t^K = \frac{r_t^K u_t - \alpha(u_t) + (1-\delta) Q_t}{Q_{t-1}}. \tag{2}$$

In each period an idiosyncratic capital quality shock, ψ_t results in a return $R_t^K \psi_t$ which is the entrepreneur's private information. Following BGG, we assume that ψ_t has a unit-mean log normal distribution that is independently drawn across time and across entrepreneurs. Specifically, $\log(\psi) \sim \mathcal{N}\left(-\frac{\sigma_\psi^2}{2}, \sigma_\psi^2\right)$. With the mean set to $-\frac{\sigma_\psi^2}{2}$, $\mathbb{E}[\psi] = 1$. σ_ψ is the period t standard deviation of $\log(\psi)$. Similarly to Christiano, Motto, and Rostagno (2014) we label σ_ψ, the cross-sectional dispersion in ψ, the risk shock and we allow it to vary stochastically over time.

Default in period t occurs when net worth becomes negative, i.e., when $N_{E,e,t} < 0$ and shock falls below a threshold $\bar{\psi}_t$ given by

$$\bar{\psi}_t = \frac{R^L_{t-1} L_{e,t-1}}{\Pi_t R^K_t Q_{t-1} K_{e,t-1}} \tag{3}$$

With the idiosyncratic shock, ψ_t drawn from a density $f(\psi_t)$ with a lower bound ψ_{min}, the probability of default is then given by

$$p(\bar{\psi}_t) = \int_{\psi_{min}}^{\bar{\psi}_t} f(\psi) d\psi$$

In the event of default the bank receives the assets of the firm and pays a proportion μ of monitoring costs to observe the realized return. Otherwise the bank receives the full payment on its loans, $R^L_t L_{e,t}/\Pi_{t+1}$ where R^L_t is the agreed loan rate at time t.

The bank's incentive compatibility constraint is now

$$\mathbb{E}_t \left[(1-\mu) R^K_{t+1} Q_t k_t \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + (1 - p(\bar{\psi}_{t+1})) \frac{R_{l,t}}{\Pi_{t+1}} l_t = R^B_{t+1} L_{e,t} \geqslant R_{t+1} L_{e,t} \right] \tag{4}$$

The left hand side part of (4) is the expected return to the bank from the contract averaged over all realizations of the shock. From (4) R^B_t is defined as

$$(1-\mu) R^K_t Q_{t-1} K_{e,t-1} \int_{\psi_{min}}^{\bar{\psi}_t} \psi f(\psi) d\psi + (1 - p(\bar{\psi}_t)) \frac{R^L_{t-1}}{\Pi_t} L_{e,t-1} = R^B_t L_{e,t-1} \tag{5}$$

In the pure BGG case we have $R^B_t = R_t = \frac{R_{n,t-1}}{\Pi_t}$, where $R_{n,t-1}$ is the nominal interest rate. In the pure GK case ψ_{min} is sufficiently high to give $p(\bar{\psi}_t) = \int_{\psi_{min}}^{\bar{\psi}_t} f(\psi) d\psi = \int_{\psi_{min}}^{\bar{\psi}_t} \psi f(\psi) d\psi = 0$. Then $R^L_t = R_{n,t}$.

Eliminating the real loan rate from (3), this becomes

$$\mathbb{E}_t \left[R^K_{t+1} Q_t K_{e,t} \left((1-\mu) \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + \bar{\psi}_{t+1} (1 - p(\bar{\psi}_{t+1})) \right) = R^B_{t+1} L_{e,t} \right] \tag{6}$$

Defining

$$\Gamma(\bar{\psi}_{t+1}) \equiv \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + \bar{\psi}_{t+1} (1 - p(\bar{\psi}_{t+1})) \tag{7}$$

$$G(\bar{\psi}_{t+1}) \equiv \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi \tag{8}$$

(6) becomes

$$\mathbb{E}_t \Big[R^K_{t+1} Q_t K_{e,t} \left[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}) \right] = R^B_{t+1} L_{e,t} \Big] \tag{9}$$

2.1.1 The Optimal Contract

The optimal contract for the risk neutral entrepreneur maximizes the average return to capital over the distribution of ψ_t taking into account the possibility of default and the cost of loans in its absence. She chooses $k_{e,t}$ and the loan rate R_t^L, which from (3) is equivalent to choosing the threshold shock ψ_{t+1}, and solves

$$\max_{\bar{\psi}_{t+1},k_{e,t}} \mathbb{E}_t\left[\left(1-\Gamma(\bar{\psi}_{t+1})\right)R_{t+1}^K Q_t k_{e,t}\right]$$

given initial net worth $n_{E,e,t}$, subject to (9) which, using (1) can be rewritten as

$$\mathbb{E}_t\Big[R_{t+1}^K Q_t k_{e,t}\left[\Gamma(\bar{\psi}_{t+1})-\mu G(\bar{\psi}_{t+1})\right]=R_{t+1}^B(Q_t k_{e,t}-n_{E,e,t})\Big] \tag{10}$$

where $k_{e,t}=K_{e,t}/P_t$ are the real capital holdings, $n_{E,e,t}=N_{E,e,t}/P_t$ the real entrepreneurial net worth and $l_{e,t}=L_{e,t}/P_t$ the real loans and P_t the price level of output.

Let λ_t be the Lagrange multiplier associated with the constraint. Then the first order conditions are

$$\begin{aligned}
k_t &: \mathbb{E}_t\left[(1-\Gamma(\bar{\psi}_{t+1})R_{t+1}^K+\lambda_t\left[(\Gamma(\bar{\psi}_{t+1})-\mu G(\bar{\psi}_{t+1}))R_{t+1}^K-R_{t+1}^B\right]\right]=0\\
\bar{\psi}_{t+1} &: \mathbb{E}_t\left[-\Gamma'(\bar{\psi}_{t+1})+\lambda_t(\Gamma'(\bar{\psi}_{t+1})-\mu G'(\bar{\psi}_{t+1}))\right]=0\\
u_t &: r_t^K=\alpha'(u_t)
\end{aligned}$$

Combining the two first order conditions, we arrive at

$$\mathbb{E}_t[R_{t+1}^K]=\mathbb{E}_t[\rho(\bar{\psi}_{t+1})R_{t+1}^B] \tag{11}$$

where the *premium on external finance*, $\rho(\bar{\psi}_{t+1})$ is given by

$$\rho(\bar{\psi}_{t+1})=\frac{\Gamma'(\bar{\psi}_{t+1})}{\left[(\Gamma(\bar{\psi}_{t+1})-\mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1})+(1-\Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1})-\mu G'(\bar{\psi}_{t+1}))\right]}.$$

2.1.2 Aggregation

We now aggregate assuming that entrepreneurs exit with fixed probability $1-\sigma_E$. To allow new entrants start up we assume exiting entrepreneurs transfer a proportion ξ_E of their wealth to new entrants. Aggregate net worth then accumulates according to

$$n_{E,t}=(\sigma_E+\xi_E)(1-\Gamma(\bar{\psi}_t))R_t^K Q_{t-1}k_{t-1}$$

and on exiting the entrepreneur consumes

$$c_{E,t}=(1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi}_t))R_t^K Q_{t-1}k_{t-1}.$$

The equilibrium is completed with the aggregate incentive compatibility constraint, assumed to be always binding and be independent from each entrepreneur type e.[5]

$$\mathbb{E}_t\left[R^K_{t+1}Q_tk_t\left[\Gamma(\bar{\psi}_{t+1})-\mu G(\bar{\psi}_{t+1})\right]\right]=\mathbb{E}_t\left[R^B_{t+1}(Q_tk_t-n_{E,t})\right].$$

2.2 Banks

At every period period there is a fixed mass of intermediaries indexed by i. Each bank allocates its funds to credit $L_{i,t}$. It funds its operations by receiving deposit from households $D_{i,t}$, emergency funding from the central bank $M_{i,t}$ and also by raising equity $N_{B,i,t}$. Each period, a fraction $1-\sigma_B$ of bankers, exit and give retain earnings to their household. An equal number of new bankers enter at the same time. They begin with a start up fund of ξ given to them by their household.

From the above specification, it follows that the bank's balance sheet is:

$$L_{i,t}=N_{B,i,t}+D_{i,t}+M_{i,t} \tag{12}$$

The bank's net worth evolves as the difference between interest income and interest expenses. Net worth of the bank accumulates in stationarized form according to :

$$(1+g_t)N_{B,i,t}=R^B_tL_{i,t-1}-R_tD_{i,t-1}-R^M_{i,t}M_{i,t-1} \tag{13}$$

To understand this dynamic problem better we can substitute for D_t from (12) and rewrite (13) as

$$(1+g_t)N_{B,i,t}=R_tN_{B,i,t-1}+(R^B_t-R_t)L_{i,t-1}-(R^M_t-R_t)M_{i,t-1} \tag{14}$$

R^M_t the interest rate of the emergency funding (LTRO) defined endogenously in the model as will be shown momentarily.

Banks exit with probability $1-\sigma_B$ per period and therefore survive for $j-1$ periods and exit in the j^{th} period with probability $(1-\sigma_B)\sigma_B^{j-1}$. Given the fact that bank pays dividends only when it exists, the banker's objective is to maximize expected discounted terminal wealth

$$V_t=\mathbb{E}_t\sum_{j=1}^{\infty}(1-\sigma_B)\sigma_B^{j-1}\Lambda_{t,t+j}n_{B,i,t+j} \tag{15}$$

subject to an incentive constraint for lenders (households) to be willing to supply funds to the banker, where $n_{B,i,t}=N_{B,i,t}/P_t$ is real net worth and P_t is the price of final output. $\Lambda_{i,t,t+j}=\beta^j\frac{\Lambda_{C,t+j}}{\Lambda_{C,t}}$ is the stochastic discount factor.

As in Gertler and Karadi (2011) there is an endogenous constraint on the banks ability to borrow is introduced. A banker after collecting deposits from households and liquidity from the central bank may divert a fraction of these funds. This occurs when the bank's

[5] This follows from (4).

value from diverting is higher than its franchise value. It is assumed that the bank can abscond a fraction $\theta \in [0,1]$ of the loans net a fraction $\theta\omega \in [0,1] < \theta$ of the central bank liquidity. The latter is is due to the high monitoring ability of the central bank to its own loanable funds. In case of absconding its funds the creditors can force the intermediary into bankruptcy at the beginning of the next period. The constraint therefore sets a limit to the bankers borrowing from either the depositors or the central bank. For the banks creditors to continue providing funds to the bank, the following incentive constraint must always hold:

$$V_t \geqslant \theta[l_{i,t} - \omega m_{i,t}] \tag{16}$$

where $l_{i,t} = L_{i,t}/P_t$ are the real loanable funds to the firms and $m_{i,t} = M_{i,t}/P_t$ the real liquidity receivable from the central bank. Bank's value must be greater or at least equal with the value of its divertable assets. When this constraint holds bankers have no incentive to steal from their creditors.

The detailed solution to the banker's problem is presented in Appendix B. In this Section we present the key equilibrium conditions of the bank's problem. Combining the optimality conditions with the banker's incentive constraint yields a central equation of the model: The leverage constraint of the bank:

$$l_{i,t} = \phi_t n_{B,i,t} + \omega m_{i,t}. \tag{17}$$

Equation (17) constraints the financial intermediary's leverage and due to this excess returns are generated. ϕ_t is the maximum adjusted leverage ratio of the bank:

$$\phi_t = \frac{\mathbb{E}_t[\Lambda_{i,t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{i,t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]}$$

and

$$\Omega_t = 1 - \sigma_B + \sigma_B \theta \phi^B_t.$$

Maximum adjusted leverage ratio depends positively on the marginal cost of the deposits and on the excess value of bank assets. As the credit spread increases, banks' franchise value V_t increases and the probability of a bank diverting its funds declines. On the other hand, as the proportion of assets that a bank can divert, θ increases, the constraint binds more.

Importantly, the maximum adjusted leverage ratio does not depend on any individual bank characteristics, therefore the heterogeneity in the bankers' holdings and net worth, does not affect aggregate dynamics. Hence, it is straightforward to express individual financial sector variables in aggregate form.

Finally, from the first order conditions yields the arbitrage condition between the lending and liquidity returns. This endogenously determines the liquidity interest rate

R_t^M:

$$\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^M - R_{t+1})] = \omega\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1}^B - R_{t+1})].$$

The excess cost to a bank of liquidity credit relative to deposits equals to the credit spread multiplied by the monitoring ability of the central bank to the liquidity provided to the bankers ω.

According to this equation, to make banks indifferent between liquidity and deposits at the margin, the central bank should set R_t^M to make the excess cost of liquidity equal to the fraction ω of the excess value of assets. Looking at the incentive constraint of the bank (16) a unit of liquidity relaxes the constraint of the banks and therefore permits a bank to expand assets by a greater amount than a unit of deposits, it is willing to pay a higher cost for this form of credit. In this way, as in Gertler and Kiyotaki (2010), the model generates an endogenously determined penalty rate for liquidity.

In steady state, the liquidity interest rate will be a convex combination of the lending rate R_t^B and the deposit interest rate R_t. In the quantitative analysis of the paper we consider the case of zero lower bound in the nominal interest rate which essentially defines the real interest -deposit- rate. Therefore, indirectly, there is a zero lower bound on the liquidity returns, since by definition are always higher than the real interest rate on deposits. The relationship between the nominal and the real liquidity rate is $R_t^M = \frac{R_{m,t-1}}{\Pi_t}$, where Π_t is the inflation.

2.2.1 Aggregation

At the aggregate level the banking sector balance sheet is:

$$l_t = n_{B,t} + m_t + d_t$$

At the aggregate level net worth is the sum of existing (old) bankers and new bankers:

$$n_{B,t} = n_{o,t} + n_{n,t}$$

Net worth of existing bankers equals earnings on assets held in the previous period net cost of deposit finance, multiplied by a fraction σ_B, the probability that they survive until the current period:

$$(1 + g_t)n_{o,t} = \sigma_B\{R_t^B l_{t-1} - R_t d_{t-1} - R_t^M m_{t-1}\}$$

Since new bankers cannot operate without any net worth, we assume that the family transfers to each one the fraction $\xi_B/(1 - \sigma_B)$ of the total value assets of exiting bankers. This implies:

$$(1 + g_t)n_{n,t} = \xi_B R_t^B l_{t-1}$$

and in aggregate bank leverage is given by

$$\phi_t^B = \frac{l_t - \omega m_t}{n_{B,t}}. \tag{18}$$

2.3 The Central Bank

The central bank can make use of two policy tools. Firstly, it adjusts the policy rate according to a Taylor monetary rule. Secondly, it supplies liquidity to the banking sector.

The relative increase in the liquidity of the banking sector is determined endogenously following the liquidity rule specified momentarily. The effectiveness of the policy comes primarily from its ability to ease the financial constraints of banks. When balance sheet constraints are tight and excess returns are positive, central bank liquidity injections loose the incentive constraint of the banks and allow to extend new lending to non-financial corporations. The easier credit conditions increase the value of capital and banks' net worth. This through a financial accelerator mechanism, increases further the banks' net worth and eases the financial constraint.

Following Gertler, Kiyotaki, and Queralto (2012), liquidity injections involve efficiency costs for the central bank: in particular, the central bank liquidity consumes resources of $\Psi_t(M_t)$, where the function Ψ_t is increasing in the quantity of liquidity provided to the banking sector. These costs could be thought as administrative costs of raising new funds through government debt or any inefficiency the central bank faces in order to provide liquidity to the banks such as identifying which banks is mostly beneficial to receive the liquidity. The function is assumed to be a quadratic function of liquidity M_t governed by the penalty parameters (τ_1, τ_2):

$$\Psi_t(M_t) = \tau_1 M_t + \tau_2 M_t^2$$

It is assumed also that the central bank turns over any profits to the treasury and receives transfers to cover any losses.

2.3.1 The Liquidity Rule

Liquidity is provided by the central bank to the banking sector according to the rule $\chi_{m,t}$, defined as the fraction of the total bank assets financed through LTRO where $\chi_{m,t} = \frac{M_t}{L_t}$. The liquidity rule $\chi_{m,t}$ responds, similarly to a conventional policy rate Taylor rule to the variables' deviations form their steady state levels. The variables we choose are: output, inflation and the lending spread.

Therefore the rule reads as follows:

$$\chi_{m,t} \quad = \quad \rho_l(\chi_{m,t-1} - \chi_{m,ss})$$

$$
\begin{aligned}
&- \quad (1-\rho_l)\kappa_m\{\theta_{\pi,l}\,(\Pi_t - \Pi) \\
&+ \quad \theta_{y,l}\,(Y_t - Y) + \theta_{dy,l}\,(Y_t - Y_{t-1}) \\
&- \quad \theta_{sp,l}\,\mathbb{E}_t[(R^K_{t+1} - R_{t+1}) - (R^{K,ss} - R^{ss})]\}
\end{aligned} \tag{19}
$$

The intensity of the liquidity intervention depends on the liquidity feedback parameter $\kappa_m \geqslant 0$. $(R^{K,ss} - R^{ss})$ is the steady state premium and Π and Y are the steady state values for output and inflation. Eliminating the responses to output and inflation changes the rule collapses to the same liquidity rule introduced in Gertler and Karadi (2011) where central bank liquidity responds only to spread deviations.

2.3.2 The Monetary Rule

The central bank sets the policy interest rate according to a Taylor Rule responding to inflation and output deviations from their steady state and also to deviations in the lending spread, in the same fashion with the liquidity rule.

The nominal interest rate is given by the following Taylor-type rule

$$
\begin{aligned}
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\Big[\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) \\
&+ \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) \\
&+ \theta_{sp} \log\left(\frac{R^K_{t+1} - R_{t+1}}{R^{K,ss} - R^{ss}}\right)\Big] + \epsilon_{MPS,t}
\end{aligned} \tag{20}
$$

The parameter θ_{sp} controls the intensity of policy rate changes to spread deviations from its steady state level and $\epsilon_{MPS,t}$ is a monetary policy shock. The link between nominal and real interest rates is given by the following Fisher relation:

$$
R_t = \frac{R_{n,t-1}}{\Pi_t}.
$$

A similar relation holds for the interest rate on liquidity.

$$
R^M_t = \frac{R_{m,t-1}}{\Pi_t}.
$$

2.4 The Government Budget Constraint

Government collects lump sum taxes T_t to finance its public expenditures G_t. We assume that the level of the government expenditures is at a fixed level relative to output (γ^G) and subject to a transitory shock g_t that follows an AR(1) process. Hence, $G_t = (\gamma^G Y_t) g_t$.

The government budget constraint thus is:

$$G = T_t + T_t^{CB}. \tag{21}$$

and the economy's resource constraint is:

$$Y_t = C_t + I_t + G_t + \Psi_t(M_t).$$

2.5 Welfare

In order to rank alternative policies we use a welfare-based criterion based on the intertemporal household expected utility:

$$\Omega_t \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} U(C_{t+\tau}, C_{t+\tau-1}, H_{t+\tau}) \tag{22}$$

In a zero-growth steady state we can write (22) in recursive form as:

$$\Omega_t = U_t + \beta \mathbb{E}_t \Omega_{t+1}. \tag{23}$$

2.6 Structural Shocks

The model is closed with eight exogenous AR(1) shock processes to technology, government spending, the real marginal cost (the latter being interpreted as a mark-up shock), the marginal rate of substitution, an investment shock, a risk premium shock, a shock to monetary policy and a risk shock. Therefore we have eight first order autoregressive processes for the variables $\{A_t, G_t, MS_t, MRSS_t, IS_t, RPS_t, MPS_t, \sigma_{\psi,t}\}$.

3 Data and Estimation

We estimate our model on quarterly data from 1991Q1 to 2018Q4 using Bayesian techniques.[6] We use a total of eight observables in the estimation. As is standard in the estimation of medium scale models, we include the real per capita growth rates of GDP, consumption, and investment, real wage growth, a measure of labour hours, the GDP deflator, and the ECB's policy rate. In order to take into account the unconventional monetary policy of the Euro Area we make use of the shadow rate by Wu and Xia (2016). This is available from 2004 onwards. For the time period 1991-2004 we use the policy rate of the ECB. Finally, we include the lending spread of the EA economy which is defined as the average lending rate minus the deposit rate.

[6] For a detailed analysis see An and Schorfheide (2007).

In our model's estimation we do not use data for liquidity injections, therefore we estimate the model as if liquidity injections were absent. We do this in order to perform our normative exercises for various liquidity rules and see which maximizes welfare.

Following the literature, some parameters of the model are calibrated to conventional values and also to match some Euro Area long term averages.

3.1 Calibration

The model's calibration is performed in order to match Euro Area stylized facts and is divided in conventional and banking parameters and it is show in Table 1. It follows broadly the calibration of the updated version of the New Area-Wide Model (NAWM), (Christoffel, Coenen, and Warne (2008), Coenen, Karadi, Schmidt, and Warne (2018)), the DSGE model of the ECB.

Banking parameter values are chosen in order to match specific Euro Area banking characteristics namely the banks' average leverage, lending spread and the bankers' planning horizon. There are three parameters that characterise the behaviour of the banking sector in the model. This is the absconding rate θ, the fraction of entering bankers initial capital fund ξ_B , and the steady-state value of the survival rate, σ_B. We calibrate these parameters to match certain steady-state moments following our data and the moments reported in Coenen et al. (2018). The steady-state leverage of the banks is set equal to 6, which corresponds to the average asset-over-equity ratio of monetary and other financial institutions as well as non-financial corporations, with weights equal to their share of assets in total assets between 1999Q1 and 2014Q4 according to the euro area sectoral accounts. The steady-state spread of the lending rate over the risk-free rate, $R^l_t - R_t$ is set to 1.656 percentage points on an annualised basis at the steady state, which is the average spread between the long-term cost of private-sector borrowing and the deposit rate for our sample period 1991Q1 to 2018Q4. The banks planning horizon is set equal to 5 years. This moment targeting exercise leads to $\theta = 0.290, \xi_B = 0.005$ and $\sigma_B = 0.942$. These parameters are also in line with the related studies in the literature. Finally, we set the monitoring parameter that the central bank has on its loanable funds to the banking sector, ω, to values from 50% - 90%. A values 50% targets a steady state bond spread half to this of the lending spread in line with Gertler and Kiyotaki (2010). In the following sections we experiment with this parameter to see its impact on our welfare results.

Entrepreneur specific parameters are the monitoring costs, their entry start up fund and their lifetime duration. We calibrate the monitoring costs in order to match an annual probability of default of 3%. This is in line with Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis (2015) where they calibrate a similar BGG-type model for the Euro Area. This target leads to monitoring costs, μ equal to 0.21 annually as assumed similarly by Christiano et al. (2014), Rannenberg (2016) and Clerc et al. (2015).

Also we set the leverage in non-financial firm sector to 2, following Rannenberg (2016); this leads to a continuity probability of the entrepreneurs equal to $\sigma_E = 0.978$. Entrepreneurs' start up fund ξ_E is set such that the external finance premium is close to the lending spread in equilibrium. Finally the idiosyncratic dispersion of the entrepreneurs σ_ψ is set to 0.27, very close to the estimate of Christiano et al. (2014) of 0.26.

The values for the share of capital α and the depreciation rate δ are chosen to 0.33 and 0.025 respectively following the estimation results of Christoffel et al. (2008). Similarly, the value of β is assigned to 0.998, chosen to be consistent with an annualised equilibrium real interest rate of 2%. Long term equilibrium values for growth, inflation target and technology are taken directly from the Euro Area data as averages of our data sample 1991Q1 to 2018Q4. Finally, the government spending as a fraction of the GDP is set to 18% also following other studies for the Euro Area.

We assume that in steady state the central bank provides zero liquidity; simulating the very low level of LTROs compared to post-2010 level provided by the ECB. Therefore we set $\chi_{m,ss}$ equal to zero. Regarding the cost of the central bank intervention, we follow Gertler et al. (2012) and we set τ_1 and τ_2 equal to 0.000125 and 0.0012 respectively corresponding to a 10 bps credit cost.

3.2 Estimation

We estimate the rest of the parameters using Bayesian techniques. We use as many observables as shocks in the model which maintains the perfect information assumption.

We treat our observable variables in order to match their data counterparts. Specifically for output, inflation, consumption and real wages we take the logarithmic first differences. For labour ours we subtract the sample mean and then divide the result with it. Interest rates and the credit spread remain unchanged. Finally for inflation we take the logarithm of it. The following measurement equations are used:

$$\text{Real GDP growth} = \log\left(\frac{Y_t}{Y_{t-1}}(1+\bar{g})\right)$$

$$\text{Real consumption growth} = \log\left(\frac{C_t}{C_{t-1}}(1+\bar{g})\right)$$

$$\text{Real investment growth} = \log\left(\frac{I_t}{I_{t-1}}(1+\bar{g})\right)$$

$$\text{Real wage growth} = \log\left(\frac{W_t}{W_{t-1}}(1+\bar{g})\right)$$

$$\text{Labour hours} = \frac{H_{d,t} - H_d^{ss}}{H_d^{ss}}$$

$$\text{Shadow interest rate} = Rn_{t-1}$$

$$\text{Inflation} = \log(\pi_t)$$

Parameter	Description	Value
A. Preferences		
β	Discount factor	0.998
B. Technology		
α	Capital share	0.670
δ	Depreciation rate	0.025
C. Banks		
θ	Banker's absconding rate	0.290
σ_B	Exit probability: bankers	0.942
ξ_B	Entry start up fund: bankers	0.005
ω	Absconding fraction for LTRO	0.500
D. Entrepreneurs		
ξ_E	Entry start up fund: entrepreneurs	0.005
σ_E	Exit probability: entrepreneurs	0.978
σ_ψ	Entrepreneur's idiosyncratic dispersion	0.2712
μ	Monitoring costs	0.2092
E. Liquidity Injections		
τ_1	Credit cost	0.000125
τ_1	Credit cost	0.0012
$\chi_{m,ss}$	Steady state liquidity level	0
F. Long Term Equilibrium		
$\bar{A}$	Steady state technology	1.000
$\bar{\pi}$	Gross inflation objective	1.005
$\bar{g}$	Steady state growth	0.003
$\frac{G}{Y}$	Gov. spending over GDP	0.180

Table 1: Calibrated Parameter Values

$$\text{Lending Spread} = \quad R_{k,t} - R_t$$

We satisfy a balanced growth path by accounting for a deterministic trend in the growth rate $\bar{g}$ in our measurement equations. We set the growth rate to the average growth for the Euro Area for the time interval we study which is $\bar{g} = 0.366\%$.

Table 2 show our priors and posterior estimates. The posterior distributions of the parameters have been estimated using the random-walk Metropolis sampler, taking into account the system priors. The estimation results are based on a Markov chain with 100000 draws. The priors for the parameters of the real economy are set in line with Smets and Wouters (2007).

Our estimates are close to those from Coenen et al. (2018), who estimate a SW model variant with financial frictions in the banking sector for the Euro Area. Most notably, we find very similar values for the monetary rule component of the model. Both estimates for the inflation coefficient, θ_π, are above 2.5, which seems to be a Euro Area characteristic in comparison with the literature on US data where these values are usually less than two. Similarities of the two models continue for the rest of the interest rate rule and the real economy parameter estimates.

			Prior		Posterior
Parameter	Description	Dist.	Mean	Std	Mean
A. Preferences					
σ_c	Relative risk aversion	$\mathcal{N}$	1.50	0.375	1.8037
ψ	Inverse Frisch elasticity	$\mathcal{N}$	2.00	0.750	1.3906
χ	Habit formation	B	0.50	0.100	0.4788
ϕ_X	Adjustment costs	$\mathcal{N}$	2.00	0.750	2.0345
B. Wage and price set.					
ξ_p	Calvo scheme: prices	B	0.50	0.100	0.5193
ξ_w	Calvo scheme: wages	B	0.50	0.100	0.6065
γ_p	Indexation: prices	B	0.50	0.100	0.3610
γ_w	Indexation: wages	B	0.50	0.100	0.4823
C. Interest-rate rule					
ρ_r	Interest-rate smoothing	B	0.75	0.100	0.7139
θ_π	Response to inflation	$\mathcal{N}$	2.00	0.250	2.6101
θ_y	Response to output gap	$\mathcal{N}$	0.12	0.050	0.0530
$\theta_{\Delta y}$	Response to $\Delta(Y_{gap})$	$\mathcal{N}$	0.12	0.050	0.2049
D. Autocorr. parameters					
ρ_A	Technology	B	0.50	0.200	0.9520
ρ_G	Gov. spending	B	0.50	0.200	0.8409
ρ_{MCS}	Marginal cost	B	0.50	0.200	0.8905
ρ_{MRSS}	Marginal rate of subst.	B	0.50	0.200	0.9565
ρ_{MPS}	Monetary policy	B	0.50	0.200	0.3579
ρ_{RPS}	Risk premium	B	0.50	0.200	0.9836
ρ_{IS}	Investment	B	0.50	0.200	0.9825
ρ_{RS}	Risk	B	0.50	0.200	0.9669
E. Shock parameters					
σ_A	Technology	Γ^{-1}	0.001	0.020	0.0064
σ_G	Gov. spending	Γ^{-1}	0.001	0.020	0.0227
σ_{MCS}	Marginal cost	Γ^{-1}	0.001	0.020	0.0068
σ_{MRSS}	Marginal rate of subst.	Γ^{-1}	0.001	0.020	0.0196
σ_{MPS}	Monetary policy	Γ^{-1}	0.001	0.020	0.0019
σ_{RPS}	Risk premium	Γ^{-1}	0.001	0.020	0.0008
σ_{IS}	Investment	Γ^{-1}	0.001	0.020	0.0295
σ_{RS}	Risk	Γ^{-1}	0.001	0.020	0.0490

Table 2: Estimation results for the model. Notes: $\mathcal{N}$ stands for the Normal distribution, B for the Betta and Γ^{-1} for the inverted Gamma distribution.

3.3 Validation

The last step in our estimation exercise is the model's validation with the first two moments of the data counterparts of the observable variables we use. Table 3 shows the results.

Our estimated model produces long run averages close to the data counterparts. It is noteworthy that we do not include any data on net worth and loans in our estimation. Nevertheless, the model provides long run averages close to the real values of the two variables. Our standard deviation estimates of the observables are also very well in line with the data. Exceptions to this are the investment growth and the variable for net worth which again we do not use in our estimation procedure.

Variable	Mean		Std	
	Data	Model	Data	Model
Output Growth	0.0037	0.0037	0.0058	0.0081
Consumption Growth	0.0032	0.0037	0.0047	0.0050
Investment Growth	0.0032	0.0037	0.0170	0.0340
Wage Growth	0.0024	0.0037	0.0038	0.0053
Labour supply	0.0000	0.0000	0.0277	0.0313
Inflation	0.0076	0.0076	0.0062	0.0050
Shadow rate	0.0121	0.0155	0.0125	0.0089
Lending spread	0.0041	0.0086	0.0010	0.0054
Net Worth*	0.0048	0.0037	0.0800	0.0489
Loans*	0.0064	0.0037	0.0116	0.0065

Table 3: Model vs Data Moment Comparison. Variables with (*) are not included in the observable variables for the estimation.

4 Steady State Analysis: Identifying the two Opposing Financial Frictions

To commence our analysis we first provide a detailed analysis on the two opposing frictions in our model. We show that an increase in liquidity benefits the economy and is welfare improving only when the positive effects from liquidity that relax the GK friction can overcome the adverse effects of the BGG friction. We outline which conditions make this result hold.

We focus on the steady state of the model and perform a steady state exercise of the model's response to changes in the liquidity provision volume.[7] The notion of liquidity provision here is a general one: we assume that χ_m,the ratio of liquidity to the total banks' asset, is deterministic and can vary in the grid [-1, 0.9].[8] When the ratio is positive, then

[7]The steady state derivations of the model are presented in Appendix D.

[8]We notice that for value of χ_m above 0.9 deposits turn negative and therefore we do not include the

liquidity is provided by the central bank authority to the banks. When the ratio is negative, the banking sector lends to the central bank. The latter case is similar to the reserves that are being deposited to the central banks' accounts. We compute the steady state equilibrium for this grid and report how our welfare measure responds along with some macro variables of interest.

Figure 1 shows the welfare, Ω_t, path according to the change in the liquidity ratio. The figure is plotted for different values of of ω, the monitoring ability of the central bank to the liquidity funds. We will show that this is an important parameter that can change the efficacy of the liquidity rule.

For the parameter $\omega = 0.5$ welfare maximizes when the liquidity ratio is about to 10% of total lending as show with the red line. Above and below of this value liquidity produces a lower welfare. With a monitoring ability of 10%, as shown by the blue line, again welfare decreases for any positive value of liquidity.

This result changes when we increase banks' monitoring parameter to a higher value, 90%, plotted under the yellow line. Welfare is increasing for higher values of liquidity provision and reaches its maximum at the end of the grid. The reasoning of why this occurs at a high level of monitoring ability is the following. An increase in the monitoring parameter increases the effectiveness of liquidity in the bank's financial constraint as shown by the banks' leverage constraint (18). For low values of monitoring ability, due to the presence of the two opposing frictions, the positive effect from the banks' constraint loosening cannot overcome the BGG friction which increases the default probability (see Figure 2). Nevertheless, a higher monitoring value such as the one plotted, increases the effectiveness of the liquidity provision and leads to a loosening of the constraint that is high enough to counteract the negative effects originating from the BGG friction and therefore more liquidity leads to higher welfare.

values above 0.9.

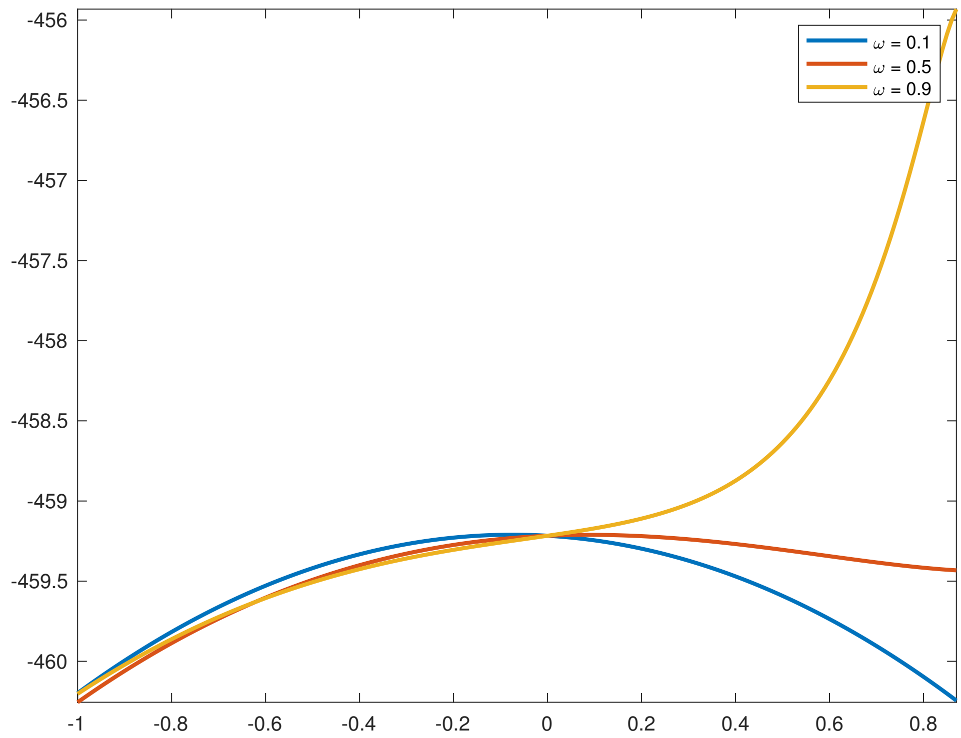

Figure 1: Welfare conditional on central bank liquidity

Figure 2 shows the paths of four important financial variables, conditional to the liquidity provision, that demonstrate the two opposing forces in the model. The two interest spreads, the external finance premium $\rho(\bar{\psi})$ and the default probability p. When the liquidity fraction turns negative, there are no significant changes in the variables' values. Let's focus on the domain of $\chi_m > 0$. As long as the liquidity provision increases, the default probability shifts upwards. At the same time, both spreads, $R^K - R$ and $R^B - R$, fall due to the loosening of the banks' financial constraint. On the contrary, the external finance premium $\rho(\psi) = R^K/R^B$ increases which leads to the increase of the default probability. This occurs because even though R^B and R^K fall, R^B falls more than R^K. These two opposing effects shed light on how the two financial frictions interact with each other. On the one hand liquidity relaxes the GK constraint and both spreads fall. On the other hand reinforces the BGG friction and the EFP together with the default probability increase.

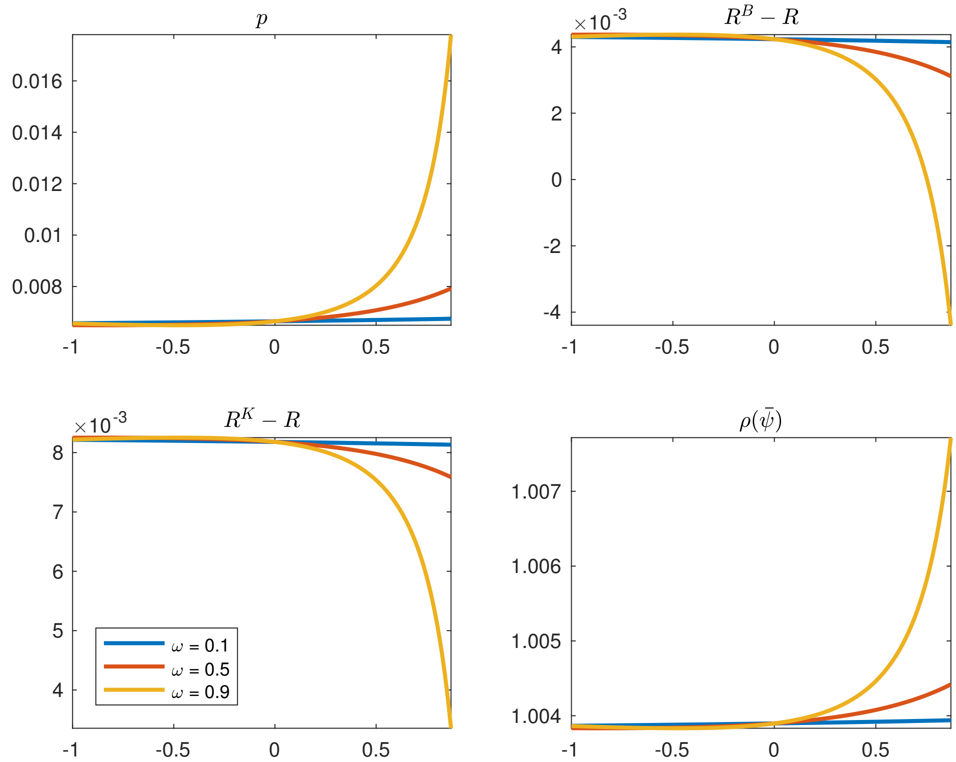

Figure 2: Financial Variables conditional on central bank liquidity

5 Welfare-Optimal Simple Rules

The concept and computation of optimized simple rules in an estimated model is central to this paper. We first make some general points before turning to the full delegation game and the results. We follow Schmitt-Grohe and Uribe (2007) quite closely, but with some important differences.

First recall the form of the estimated nominal interest rate rule: The nominal interest rate is given by the following Taylor-type rule

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\Big[\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) + \theta_{sp} \log\left(\frac{R^K_{t+1} - R_{t+1}}{R^{K,ss} - R^{ss}}\right)\Big] + \epsilon_{MPS,t} \tag{24}$$

Unlike rules studied in the NK literature which respond to the output gap and therefore a flexi-price version of the model, this rule makes no such demands on the policymaker and rational agents; it only requires knowledge of the model itself and its deterministic steady state. Schmitt-Grohe and Uribe (2007) refer to such rules as 'implementable'.

For optimal policy purposes we remove the monetary policy shock $\log(MPS_t)$ and re-parameterize the rule as the nominal interest rate is given by the following Taylor-type rule

$$\begin{aligned} \log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_y \log\left(\frac{Y_t}{Y}\right) + \alpha_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right) \\ &+ \alpha_{sp} \log\left(\frac{R^K_{t+1} - R_{t+1}}{R^{K,ss} - R^{ss}}\right) \end{aligned} \tag{25}$$

which allows for the possibility of an integral rule with $\rho_r = 1$ Let $\rho \equiv [\rho_r, \alpha_\pi, \alpha_y, \alpha_{dy}]$ be the policy choice of feedback parameters that defines the exact form of the rule. We restrict ourselves to a class of possible rules that are locally saddle-path stable in the vicinity of the non-stochastic (deterministic) steady state. We denote this sub-set of rules by S; thus $\rho \in S$. Similarly we re-parameterize the liquidity rule (19).

We begin by defining the inter-temporal household welfare at time t in recursive Bellman stationarized form in a symmetric equilibrium form as:

$$\Omega_t = U_t(C_t, C_{t-1}, H^s_t) + \beta_g \mathbb{E}_t\left[\Omega_{t+1}\right] \tag{26}$$

where β_g is a growth-adjusted discount factor defined by $\beta_g \equiv \beta(1+g)^{1-\sigma}$.

Optimal monetary policy at time $t = 0$, sets steady-state values for the nominal interest rate instrument $R_{n,t}$, denoted by R_n given initial values for the predetermined variables z_0, solves the maximization problem :

$$\max_{\rho \in S} \Omega_0(\mathsf{z}_0, R_n, \rho) \tag{27}$$

In fact the long-run (steady state) gross inflation rate target in the rule which we take to be $\Pi \geqslant 1$ (ruling out a liquidity trap) uniquely pins down the rest of the steady state so we can rewrite (27) as

$$\max_{\rho \in S} \Omega_0(\mathsf{z}_0, \Pi, \rho) \tag{28}$$

But this is a *conditional* and *time-inconsistent criterion* as the optimized rule at time t becomes

$$\max_{\rho \in S} \Omega_t(\mathsf{z}_t, \Pi, \rho) \Rightarrow \rho = \rho(\mathsf{z}_t, \Pi) \tag{29}$$

and there emerges an *incentive to re-optimize.*

We remove one source of time-inconsistency by choosing a welfare conditional on being at the steady state $\mathsf{z}_t = \mathsf{z}$, which is policy-invariant, and the choice of Π which is a policy choice.[9]. The optimization problem then becomes

$$\max_{\rho \in S} \Omega_t(\mathsf{z}, \Pi, \rho) \Rightarrow \rho = \rho(\mathsf{z}, \Pi) \tag{30}$$

Since z is policy-invariant, in what follows we simply write $\rho = \rho(\Pi)$ Thus welfare at the steady state is maximized *on average* over all realizations of the shocks driving the exogenous stochastic processes give their deterministic steady states.[10] The optimal ρ^* is computed using a *second-order perturbation solution* [11] But there are *no ZLB considerations* for the nominal interest rate as yet. This leads us to the delegation game.

6 The Delegation Game

We now turn to ZLB considerations for the nominal interest rate rule following the methodology set out in Deak, Levine, and Pham (2020). We examine the solution of the three-stage delegation game in the estimated model in the case where the choice of response parameters ρ for both monetary and liquidity policy is delegated to a central bank with a 'modified' objective of the form (31) where U_t is household utility and the rule takes the form (24). The equilibrium of this ZLB delegation mandate is solved by backward induction in the following three-stage delegation game.

1. **Stage 1**: The policymaker chooses a per period probability of hitting the ZLB and designs the optimal loss function in the mandate.

2. **Stage 2**: The optimal steady state inflation rate consistent with stage 1 is chosen.

3. **Stage 3**: The CB receives the mandate in the form of a modified purely stochastic welfare criterion of the form $\Omega_t(\mathsf{Z}_t, \Pi, \rho)$ of the form (26) with an additional penalty to limit the variance of the nominal interest rate rule. Welfare is then optimized with respect to $\rho \in S$ resulting in an optimized simple rule.

This delegation game is solved by backwards induction as follows:

6.1 Stage 3: The CB Choice of Rule

Given a steady state inflation rate target, Π, the Central Bank (CB) receives a mandate to implement the rule (24) and to maximize with respect to $\rho \in S$ a modified welfare

[9] This follows Schmitt-Grohe and Uribe (2007) and is the *timeless* criterion proposed by Woodford (2003), Chapter 7, based on Levine and Currie (1987)

[10] The maximization of the unconditional welfare under exogenous uncertainty can be compared with the optimal strategy for the board game backgammon whose outcome depends on throws of dice as well as skill. This contrasts with deterministic games such as chess.

[11] This is implemented in a Dynare program that calls a matlab subroutine **fminconc** that finds a constrained minimum of a function of several variables. A general toolkit for any DSGE model set-up is available for this.

criterion

$$\begin{aligned}\Omega_t^{mod} &\equiv \mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^\tau\left(U_{t+\tau}-w_r\left(R_{n,t+\tau}-R_n\right)^2\right)\right]\\ &= \left(U_t-w_r\left(R_{n,t}-R_n\right)^2\right)+\beta(1+g)^{1-\sigma}\mathbb{E}_t\left[\Omega_{t+1}^{mod}\right] \qquad (31)\end{aligned}$$

One can think of this as a mandate with a penalty function $P = w_r\left(R_{n,t}-R_n\right)^2$, penalizing the variance of the nominal interest rate with weight w_r.[12]

Following Den Haan and Wind (2012), an alternative mandate that only penalizes the zero interest rate in an asymmetric fashion is $P = P(a_t)$ where the occasionally binding constraint is $a_t \equiv R_{n,t} - 1 \geqslant 0$ with

$$P = P(a_t) = \frac{\exp(-w_r a_t)}{w_r} \qquad (32)$$

and chooses a large w_r. $P(a_t)$ then has the property

$$\begin{aligned}\lim_{w_r\to\infty} P(a_t) &= \infty \text{ for } a_t < 0\\ &= 0 \text{ for } a_t > 0\end{aligned}$$

Thus $P(a_t)$ enforces the ZLB approximately but with more accuracy as w_r becomes large. Stages 3–1 then proceed as before, but we now confine ourselves to a large w_r which will enable Π to be close to unity.

Both the symmetric and asymmetric forms of a ZLB mandate result in a probability of hitting the ZLB

$$p = p(\Pi, \rho^*(\Pi, w_r)) \qquad (33)$$

where $\rho^*(\Pi, w_r)$ is the optimized form of the rule given the steady state target Π and the weight on the interest rate volatility, w_r.

6.2 Stage 2: Choice of the Steady State Inflation Rate Π

Given a target low probability $\bar{p}$ and given w_r, $\Pi = \Pi^*$ is chosen so satisfy

$$p(R_{n,t} \leqslant 1) \equiv p(\Pi^*, \rho^*(\Pi^*, w_r)) \leqslant \bar{p} \qquad (34)$$

This then achieves the ZLB constraint

$$R_{n,t} \geqslant 1 \text{ with high probability } 1-\bar{p} \qquad (35)$$

[12] This closely follows the approximate form of the ZLB constraint of Woodford (2003) and Levine, McAdam, and Pearlman (2008).

where $R_{n,t}$ is the nominal interest rate.

6.3 Stage 1: Design of the Mandate

The policymaker first chooses a per period probability $\bar{p}$ of the nominal interest rate hitting the ZLB (which defines the tightness of the ZLB constraint). Then it maximizes the actual household intertemporal welfare

$$\Omega_t = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} U_{t+\tau} \right] = U_t + \beta(1+g)^{1-\sigma} \mathbb{E}_t \left[\Omega_{t+1} \right] \tag{36}$$

with respect to w_r.

This three-stage delegation game defines an equilibrium in choice variables w_r^*, ρ^* and Π^* that maximizes the true household welfare subject to the ZLB constraint (35).

In principle a similar constraint applies to the liquidity rule and the constraint that $\chi_{m,t} \in [-1, 1]$. But since the optimal steady state studied in Section 4 is close to zero and far from a boundary the probability of hitting an upper or lower bound is very small and can be ignored for the liquidity rule.

7 Optimized Liquidity and Interest Rate Rule

This Section presents the main results of our work. Firstly, we provide our estimations for an optimized liquidity rule alongside an optimized monetary rule subject to the ZLB constraint that maximizes welfare. Secondly, we decompose the rule and study what is the welfare gains by considering only separate rule components at the same time. We then proceed to finding the determinacy regions for the parameters in the liquidity rule. Lastly, we assess the impulse responses of the optimized rules to a risk shock and a monetary policy shock compared to the estimated model without the liquidity rule.

7.1 Liquidity Rule and Welfare Optimization

The liquidity rule is given by (19) which we re-parameterize as for the monetary rule as

$$\begin{aligned} \chi_{m,t} &= \rho^l(\chi_{m,t-1} - \chi_{m,ss}) - \kappa_m \{ \alpha_\pi^l (\Pi_t - \Pi) \\ &+ \alpha_l^l (Y_t - Y) + \alpha_{dy}^l (Y_t - Y_{t-1}) \\ &- \alpha_{sp}^l \mathbb{E}_t[(R_{t+1}^K - R_{t+1}) - (R^{K,ss} - R^{ss})] \} \end{aligned} \tag{37}$$

The Optimized simple rule (with ZLB)															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.764	4.045	1.275	4.871	23.303	0.999	12.931	0.014	0.254	37.903	36.590	1.008	-472.148	-0.053	0.010	27
0.980	3.415	3.751	5.048	22.899	0.999	13.529	0.024	0.009	34.862	34.777	1.007	-472.088	-0.037	0.025	5
0.821	4.175	3.049	4.053	22.724	0.999	13.307	0.011	0.003	34.867	34.465	1.005	-472.007	-0.015	0.050	3
The Optimized simple rule (without ZLB)															
ρ_r^*	α_π^*	α_y^*	α_{dy}^*	α_{sp}^*	ρ_r^{l*}	α_π^{l*}	α_y^{l*}	α_{dy}^{l*}	α_{sp}^{l*}	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.937	3.618	3.049	4.1939	23.148	0.997	12.998	0.007	0.002	35.687	34.060	1.000	-471.949	0.000	0.150	0
0.736	19.857	0.109	6.225	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	-472.065	-0.031	0.089	0
Estimated model															
ρ_r^*	$\frac{\alpha_\pi^*}{1-\rho_r^*}$	$\frac{\alpha_y^*}{1-\rho_r^*}$	$\frac{\alpha_{dy}^*}{1-\rho_r^*}$	$\frac{\alpha_{sp}^*}{1-\rho_r^*}$	ρ_r^{l*}	$\frac{\alpha_\pi^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_y^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_{dy}^{l*}}{1-\rho_r^{l*}}$	$\frac{\alpha_{sp}^{l*}}{1-\rho_r^{l*}}$	κ_m	Π^*	Ω^*	CEV (%)	p_zlb	w_r^*
0.713	2.610	0.053	0.204	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.007	-472.337	-0.103	0.007	-
0.713	2.610	0.053	0.204	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	-472.200	-0.066	0.101	-
0.713	2.610	0.053	0.204	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	-472.194	-0.065	0.100	-

Table 4: Welfare Analysis

Given a particular equilibrium for C_t and H_t and single-period utility, $U_t = U(C_t, C_{t-1}, H_t)$ we then compute CE_t, the increase in the given by a 1% increase in consumption, by defining the variable:

$$\begin{aligned} CE_t &\equiv U_t(1.01\,C_t, 1.01\,C_{t-1}/(1+g), H_t) - U_t \\ &+ \mathbb{E}_t\left[(1+g_{t+1})\beta_{g,t+1}CE_{t+1}\right] \end{aligned}$$

Then we use the deterministic steady state of CE_t, CE, to compare welfare outcomes: for two welfare outcomes. For Table 4 all outcomes are measured relative to the best available rule which is the one without a ZLB constrain on monetary policy with the trend (steady state) net inflation at zero ($\Pi = 1$). Thus the consumption equivalent variation (CEV) as well as ranking different rules measures the welfare cost of imposing the ZLB the constraint.

Two results stand out from Table 4: first, for the case of optimzed rules without ZLB considerations, we find a welfare-optimized combination of rules where the welfare benefits of the liquidity rule outweigh negative effects. Second, both the monetary and liquidity rules involve a strong response to the interest rate spread.

7.2 Liquidity Rules Decomposition and Welfare

In this section we provide a decomposition of the rule to its different components and report the welfare value for each of the different specifications. Specifically we look for the consumption equivalence variations when we activate only one component at a time, or a combination of the components, and turn off the rest. We ought to provide a more straightforward policy recommendation on which rule specification increases welfare comparing our model with the liquidity rule to our estimated model without the rule. We find that when we consider zero lower bound in the model our liquidity rule increases welfare when the rule responds to all components. The same holds for the case of no zero lower bound considerations. Deviations from the full rule either provide indeterminacy or large welfare costs.

For W^j_{policy} and W_{no_policy}, we now define $CE \equiv \frac{W^j_{policy} - W_{no\_policy}}{CE}$ reported in Table below. Here W_{no_policy} is the stochastic mean of welfare when the liquidity rule is inactive and W^j_{policy} the stochastic mean of welfare when a component j of the liquidity rule is active.

We do this exercise for the case of considering the zero lower bound and for the case that we do not. We assume that the monitoring ability of the central bank to the banks' liquidity is equal to 90%. In Appendix F we show that results are qualitatively the same with a more moderate value of ω equal to 50%, the value used by Gertler and Kiyotaki (2010).

Table 5 shows our results for the case of a probability 0.01 to hit the zero lower bound. When we consider all the components of the rule, then the rule increases welfare compared to the estimated model. When the rule responds only to credit spread or output changes, then there is indeterminacy. For the cases where our rule responds only to inflation and inflation together with output there are large welfare losses. These losses indicate that clearly these rule specifications are not the correct ones for the policymaker to employ. Table 6 shows the same exercise when there is no a ZLB constraint in the model. The results are qualitative similar to the ZLB case with the welfare that responds to all variables being the one that increases welfare.

Rule Targets	Welfare Value	Consumption Eq. Change
Estimated	-472.347	0
Spread	Indeterminacy	-
Inflation	-513.458	-10.985
Output	Indeterminacy	-
Inf. + Output	-484.244	-3.179
All	-472.148	0.053

Table 5: Welfare changes under zero lower bound. Notes: probability of hitting the zero lower bound: 1%; optimal inflation:1.008%

7.3 Determinacy Analysis of the Liquidity Rule

The following Figure depicts the determinacy regions for different pairs of the liquidity rule parameters; $\{\theta_{y,l}, \theta_{\pi,l}\}$, $\{\theta_{y,l}, \theta_{sp,l}\}$, $\{\theta_{\pi,l}, \theta_{sp,l}\}$. We choose a grid for each parameter in a neighbourhood around the optimized values of the liquidity rule parameters found

Rule Targets	Welfare Value	Consumption Eq. Change
Estimated	-472.194	0
Spread	Indeterminacy	-
Inflation	-511.365	-10.470
Output	Indeterminacy	-
Inf. + Output	-510.393	-10.211
All	-471.949	0.067

Table 6: Welfare changes under the assumption of no zero lower bound

in our optimization exercise in the previous subsection. The remaining parameters of the model are set to their estimated value.

Results are shown in Figure 3. The light grey area depicts the determinacy region where the darker area the regions were no determinacy occurs. Nota that responses of output and inflation enter with a negative sign in our liquidity rule while the response to spread with a positive sign. Therefore a positive value for $\{\theta_{y,l}, \theta_{\pi,l}\}$ translates to a negative response of the rule. Part a. of Figure 3 shows the couple $\theta_{y,l}$ and $\theta_{\pi,l}$. As expected, shows that determinacy is achieved when both parameters take values that close and larger than zero. Indeterminacy arises for all negative values of the pair with an exception of low negative values of $\theta_{y,l}$ and $\theta_{\pi,l}$. Part b. of the Figure shows the determinacy regions for the couples $\theta_{y,l}$ and $\theta_{sp,l}$. For all the parameter values of the spread coefficient, when the output coefficient remains greater than minus one, determinacy is obtained. Finally part c. shows the couple $\theta_{sp,l}\}$ and $\{\theta_{\pi,l}$. For the biggest part, determinacy is achieved for positive values of the inflation parameter and values of the spread parameter that exceed thirty. For values of the spread parameter less than thirty, the inflation coefficient needs to go into the negative territory.

7.4 Determinacy Analysis of the Interest Rate Rule

Figure 4 shows the determinacy regions for different pairs of the same feedback parameter as in the previous subsection, this time for the interest rate rule. Namely: $\{\theta_y, \theta_\pi\}$, $\{\theta_y, \theta_{sp}\}$, $\{\theta_\pi, \theta_{sp}\}$. Again, the grid of the parameter values is in a neighbourhood around the optimized values of the interest rate rule. The remaining parameters of the model are set to their estimated value. The light grey area depicts the determinacy region where the darker area the regions were no determinacy occurs. In the case of the interest rate rule, determinacy is achieved for all values and all combinations of the parameters. This is at the same time where liquidity rule is also active.

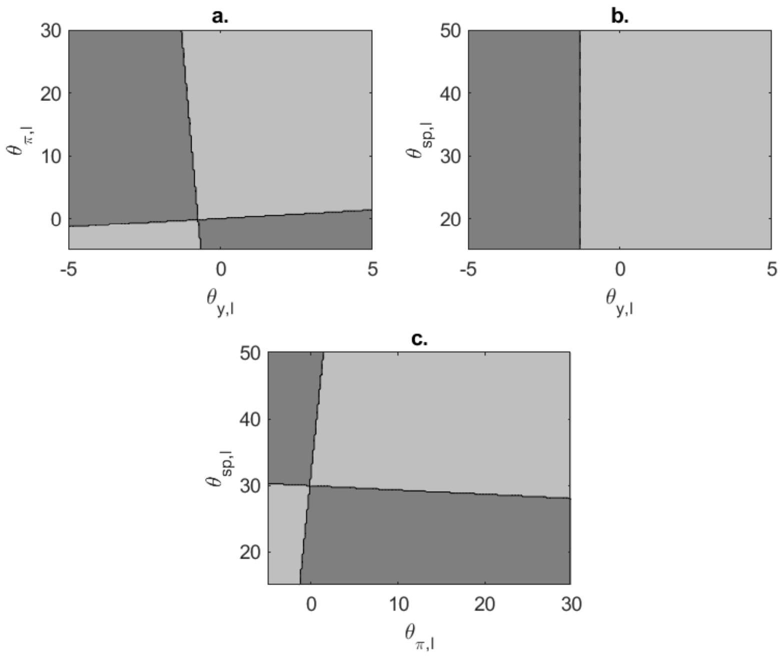

Figure 3: Determinacy Analysis for the liquidity rule feedback parameters. Notes: Remaining parameters at their estimated values. The darker area corresponds to the indeterminacy region while the light grey in the determinacy region

7.5 Impulse Responses

Figures 5 and 6 display the dynamic responses of various variables to an unanticipated shock in risk, similarly to Christiano et al. (2014), and the monetary policy shock. We consider two model versions. The first is our estimated model without a liquidity rule and with the monetary rule taking its estimated values. The second specification is the estimated model when both a liquidity and a monetary rule are at work taking their optimized values. The blue line shows the responses under no policy intervention, labelled as "Estimated Model" while the yellow dotted line shows the responses of the model when we consider our optimized policy rules. The optimized coefficients for the monetary and liquidity rule are set to be the ones that we find welfare optimizing (shown in the next section).

A shock in the idiosyncratic dispersion of an entrepreneur in our estimated model specification, as plotted in Figure 5, produces a sharp increase in the default probability of the entrepreneurs. Credit spread charged by banks increases and banks provide less credit leading to a reduction in investment and output. In our estimated model, there is a policy rate drop to accommodate the fall in output and inflation which makes them gradually return back to their steady state levels.

A liquidity rule that responds to all three variables' deviations, namely output, inflation and spread changes, manages to keep the economy close to the steady state level and

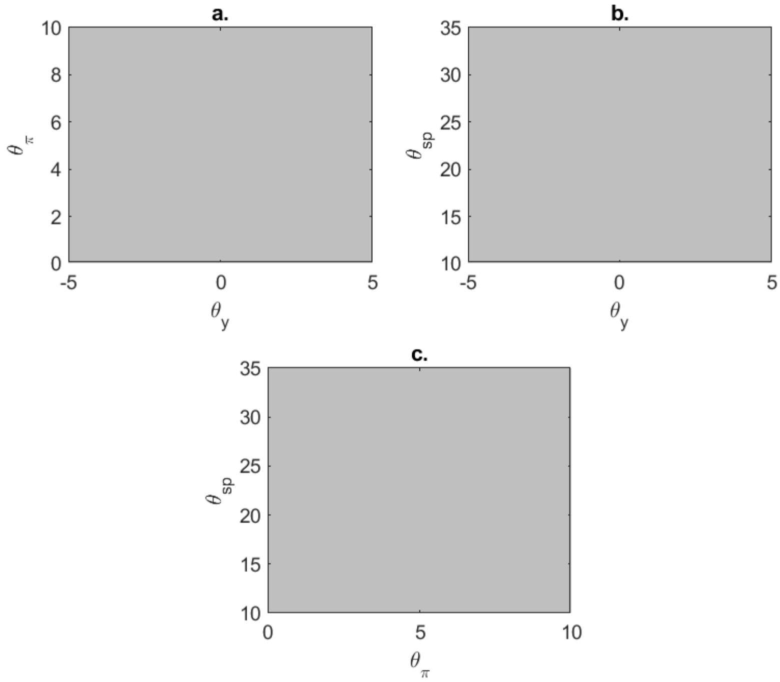

Figure 4: Determinacy Analysis for the interest rate rule feedback parameters. Notes: Remaining parameters at their estimated values. The darker area corresponds to the indeterminacy region while the light grey in the determinacy region

alleviate the negative consequences of the shock. Although there is not much of a change in the default probabilities compared to the estimated model, investment and thus output do increase due to the stabilizing forces of the two rules. The credit spread remains almost in its steady state value, similarly to the price of capital. This reduces bank profitability due to the lower spreads leading to a reduction in banks' net worth compared to the estimated specification.

Figure 6 shows the responses to a contractionary monetary policy shock. In our baseline specification without the liquidity or the optimized interest rate rule, the shock has the well documented effects on our variables of interest. It is contractionary, banking spread increases and so does the default probability of the firms. When we activate the optimized policy for both rules, the effects from the shock are almost muted. This is due to the optimized interest rate rule. To show this we also plot the responses when only the optimized liquidity rule is activated and the policy rule takes its estimated values without responding to spreads.

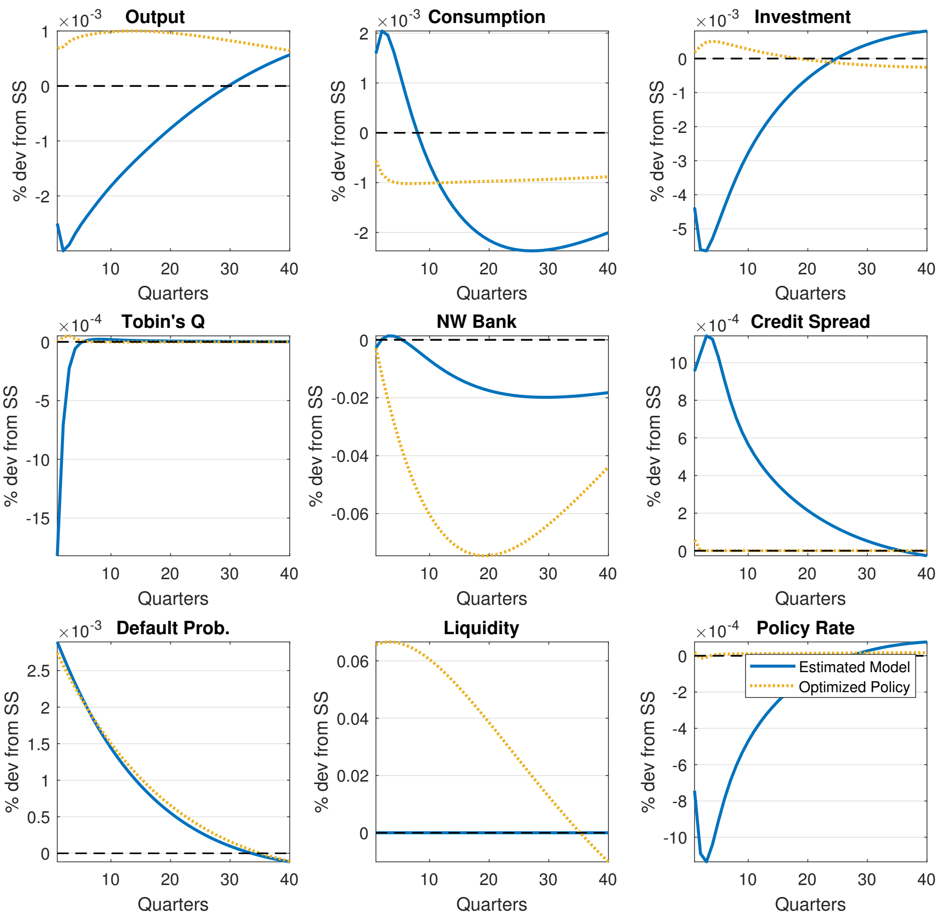

Figure 5: Impulse Responses to a Risk Shock.

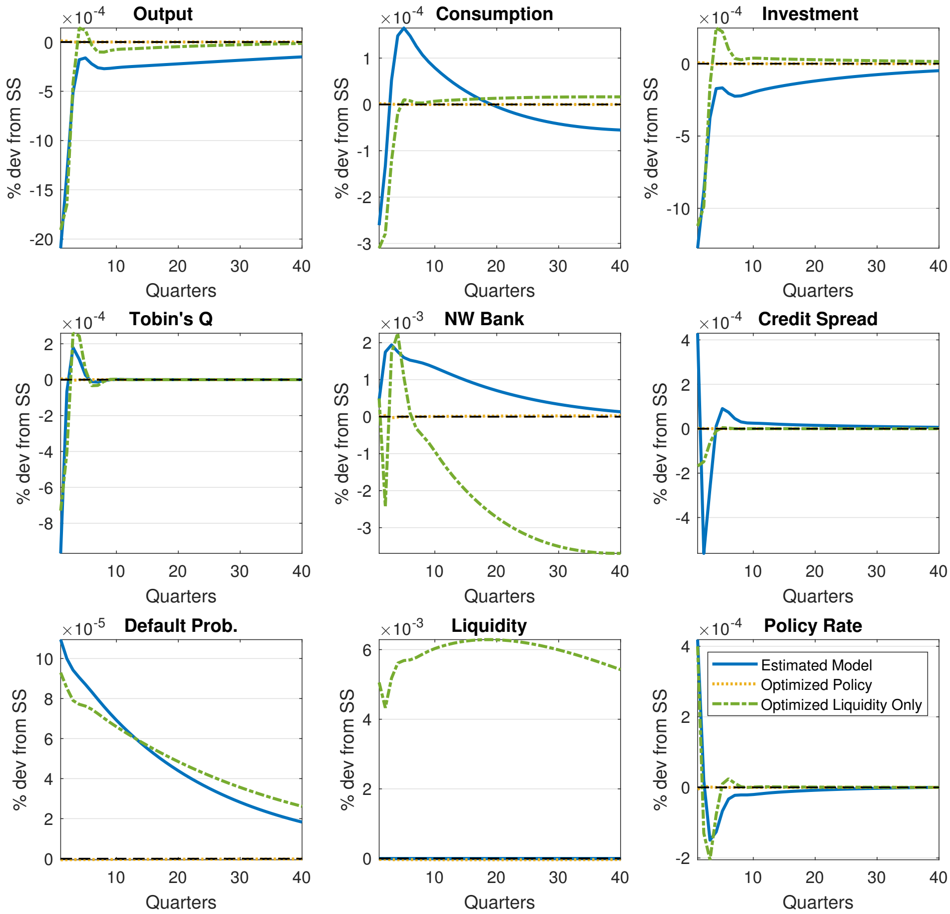

Figure 6: Impulse Responses to a Monetary Policy Shock.

8 Conclusions

This paper has employed a medium-sized NK model estimated by Bayesian methods to study a combination of interest rate and liquidity rules. The novel feature of the model is the combination of two financial frictions, one for the bank-household side and one for the bank-firm side. These are modelled using the frameworks of Gertler and Kiyotaki (2010) and Bernanke et al. (1999) respectively. The motivation for including both these features is that the implementation of the liquidity rule is welfare-enhancing for the first of these frictions but welfare-reducing for the second. The reason for this are that on the household side liquidity injections by the central bank bypasses the financial friction, but on the firm side increases the probability of default by firms.

Our main results are first, we find a welfare-optimized combination of rules where the welfare benefits of the liquidity rule outweigh negative effects. Second both the monetary and liquidity rules involve a strong response to the interest rate spread. Third, If the policymaker ignores some components of the liquidity rule the outcome can be very welfare-reducing and even indeterminate.

The focus of our paper the interaction of conventional monetary policy and liquidity injections, but our general framework and methodology is well-suited for other dimensions of policy. This could include macro-prudential and fiscal policy rules and will be the focus of our future research agenda

References

Acharya, V. V., T. Eisert, C. Eufinger, and C. Hirsch (2019): "Whatever it takes: The real effects of unconventional monetary policy," *The Review of Financial Studies*, 32, 3366–3411.

Adrian, T. and H. S. Shin (2010): "Chapter 12 - Financial Intermediaries and Monetary Economics," Elsevier, vol. 3 of *Handbook of Monetary Economics*, 601 – 650.

Allen, F. and D. Gale (2000): "Bubbles and crises," *The Economic Journal*, 110, 236–255.

Altunbas, Y., L. Gambacorta, and D. Marques-Ibanez (2010): "Bank risk and monetary policy," *Journal of Financial Stability*, 6, 121–129.

An, S. and F. Schorfheide (2007): "Bayesian analysis of DSGE models," *Econometric Reviews*, 26, 113–172.

Angelini, P., S. Neri, and F. Panetta (2014): "The interaction between capital requirements and monetary policy," *Journal of money, credit and Banking*, 46, 1073–1112.

Bernanke, B. and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79, 14–31.

Bernanke, B., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in Quantitative Business Cycles," in *Handbook of Macroeconomics, vol. 1C*, ed. by M. Woodford and J. B. Taylor, Amsterdam: Elsevier Science.

Bocola, L. (2016): "The Pass-Through of Sovereign Risk," *Journal of Political Economy*, 124, 879–926.

Buch, C. M., S. Eickmeier, and E. Prieto (2014): "In search for yield? Survey-based evidence on bank risk taking," *Journal of Economic Dynamics and Control*, 43, 12–30.

Cahn, C., J. Matheron, and J.-G. Sahuc (2017): "Assessing the Macroeconomic Effects of LTROs during the Great Recession," *Journal of Money, Credit and Banking*, 49, 1443–1482.

Calvo, G. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

Christiano, L., M. Eichenbaum, and C. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113, 1–45.

Christiano, L. J., R. Motto, and M. Rostagno (2014): "Risk shocks," *The American Economic Review*, 104, 27–65.

Christoffel, K. P., G. Coenen, and A. Warne (2008): "The new area-wide model of the euro area: a micro-founded open-economy model for forecasting and policy analysis," .

Clerc, L., A. Derviz, C. Mendicino, S. Moyen, K. Nikolov, L. Stracca, J. Suarez, and A. Vardoulakis (2015): "Capital regulation in a macroeconomic model with three layers of default," 40.

Coenen, G., P. Karadi, S. Schmidt, and A. Warne (2018): "The New Area-Wide Model II: an extended version of the ECB's micro-founded model for forecasting and policy analysis with a financial sector," *ECB Working Paper, No 2200.*

Curdia, V. and M. Woodford (2011): "The central-bank balance sheet as an instrument of monetarypolicy," *Journal of Monetary Economics*, 58, 54–79.

Deak, S., P. Levine, and S. T. Pham (2020): "Mandates and Monetary Rules a New Keynesian Framework," School of Economics Discussion Papers 0120, University of Surrey.

Delis, M. D., I. Hasan, and N. Mylonidis (2017): "The Risk-Taking Channel of Monetary Policy in the US: Evidence from Corporate Loan Data," *Journal of Money, Credit and Banking*, 49, 187–213.

Dell'Ariccia, G., L. Laeven, and G. A. Suarez (2017): "Bank Leverage and Monetary Policy's Risk-Taking Channel: Evidence from the United States," *The Journal of Finance*, 72, 613–654.

Den Haan, W. and J. D. Wind (2012): "Nonlinear and Stable Perturbation Based Approximation," *Journal of Economic Dynanamics and Control*, 36, 1477–1479.

Diamond, D. W. and R. G. Rajan (2012): "Illiquid banks, financial stability, and interest rate policy," *Journal of Political Economy*, 120, 552–591.

Dixit, A. K. and J. E. Stiglitz (1977): "Monopolistic Competition and Optimal Product Diversity," *American Economic Review*, 67, 297–308.

Eggertsson, G. and M. Woodford (2003): "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, No. 1, 139–211.

Ferrero, A., R. Harrison, and B. Nelson (2018a): "Concerted efforts? Monetary policy and macro-prudential tools," .

——— (2018b): "House price dynamics, optimal ltv limits and the liquidity trap," .

Gertler, M. and S. Gilchrist (2018): "What Happened: Financial Factors in the Great Recession," Tech. rep., NBER Working Paper.

Gertler, M. and P. Karadi (2011): "A model of unconventional monetary policy," *Journal of Monetary Economics*, 58, 17–34.

Gertler, M. and N. Kiyotaki (2010): "Financial intermediation and credit policy in business cycle analysis," *Handbook of Monetary Economics*, 3, 547–599.

Gertler, M., N. Kiyotaki, and A. Queralto (2012): "Financial Crises, Bank Risk Exposure and Government Financial Policy," *Journal of Monetary Economics*, 59, S17–S34.

Ioannidou, V., S. Ongena, and J.-L. Peydró (2014): "Monetary policy, risk-taking, and pricing: Evidence from a quasi-natural experiment," *Review of Finance*, 19, 95–144.

Jiménez, G., S. Ongena, J.-L. Peydró, and J. Saurina (2014): "Hazardous Times for Monetary Policy: What Do Twenty-Three Million Bank Loans Say About the Effects of Monetary Policy on Credit Risk-Taking?" *Econometrica*, 82, 463–505.

Joyce, M., D. Miles, A. Scott, and D. Vayanos (2012): "Quantitative easing and unconventional monetary policy–an introduction," *The Economic Journal*, 122, F271–F288.

King, R., C. Plosser, and S. Rebelo (1988): "Production, Growth and Business Cycles I: The basic Neoclassical Model," *Journal of Monetary Economics*, 21, 195–231.

Kiyotaki, N. and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211 – 248.

Laureys, L. and R. Meeks (2018): "Monetary and macroprudential policies under rules and discretion," *Economics Letters*, 170, 104–108.

Levine, P. and D. Currie (1987): "The Design of Feedback Rules in Linear Stochastic Rational Expectations Models," *Journal of Economic Dynamics and Control*, 11, 1–28.

Levine, P., P. McAdam, and J. Pearlman (2008): "Quantifying and Sustaining Welfare Gains from Monetary Commitment," *Journal of Monetary Economics*, 55, 1253–1276.

Lown, C. and D. P. Morgan (2006): "The credit cycle and the business cycle: new findings using the loan officer opinion survey," *Journal of Money, Credit and Banking*, 1575–1597.

Maddaloni, A. and J.-L. Peydró (2011): "Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the Euro-area and the US lending standards," *The Review Of Financial Studies*, 24, 2121–2165.

Rannenberg, A. (2016): "Bank Leverage Cycles and the External Finance Premium," *Journal of Money Credit and Banking*, 48, 1477–1479.

Schmitt-Grohe, S. and M. Uribe (2007): "Optimal Simple and Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics*, 54, 1702–1725.

Sims, E. and J. C. Wu (2021): "Evaluating central banks tool kit: Past, present, and future," *Journal of Monetary Economics*, 118, 135–160.

Smets, F. and R. Wouters (2007): "Shocks and frictions in US business cycles: A Bayesian DSGE approach," *American Economic Review*, 97, 586–606.

Tsiaras, S. (2018): "Financial Crisis, Monetary Base Expansion and Risk," Tech. rep., School of Economics, University of Surrey.

van der Kwaak, C. (2017): "Financial Fragility and Unconventional Central Bank Lending Operations," *Unpublished Manuscript*.

Woodford, M. (2003): *Interest and Prices. Foundations of a Theory of Monetary Policy*, Princeton University Press.

Wu, J. C. and F. D. Xia (2016): "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money Credit and Banking*, 48, 253–291.

9 Appendix

The structure of the Appendix is as follows. In Section A, we lay out the standard NK model without financial frictions. In the end of the section we provide the full standard NK model listing. In Section B, we present in detail the solution of the banker's problem. Our model combines the standard NK presented here with our financial frictions framework. The financial frictions system of equations is presented in Section C.

A The Core NK Model without a Banking Sector

We now develop an NK model with a stationarized RBC model at its core. Now we add sticky prices and nominal wages. The household sector and its supply of homogeneous is as in the RBC core. The only difference with the textbook NK model is that households invest in bank deposits instead of bonds which is usually the investment vehicle in the

NK model. We therefore focus on the supply side and the modelling of price and wage stickiness.

A.1 Households

We choose preferences compatible with balanced growth (see King, Plosser, and Rebelo (1988)). With external habit in consumption, household j has a single-period utility

$$U_t^j = \frac{(C_t^j - \chi C_{t-1}/(1+g_t))^{1-\sigma_c} \exp\left(\frac{(\sigma_c-1)(H_t^j)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c}; \quad \chi \in [0,1) \quad \sigma_l > 0$$
$$\to \log(C_t^j - \chi C_{t-1}/(1+g_t)) - \frac{(H_t^j)^{1+\sigma_l}}{1+\sigma_l} \text{ as } \sigma_c \to 1$$

where C_{t-1} is aggregate per capita consumption whereas with internal habit we have

$$U_t^j = \frac{(C_t^j - \chi C_{t-1}^j/(1+g_t))^{1-\sigma_c} \exp\left(\frac{(\sigma_c-1)(H_t^j)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c}; \quad \chi \in [0,1) \quad \sigma_l > 0$$
$$\to \log(C_t^j - \chi C_{t-1}^j/(1+g_t)) - \frac{(H_t^j)^{1+\sigma_l}}{1+\sigma_l} \text{ as } \sigma_c \to 1$$

Defining an instantaneous marginal utility by

$$U_{C,t} = (C_t - \chi C_{t-1}/(1+g_t))^{-\sigma_c} \exp\left(\frac{(\sigma_c-1)H_t^{1+\sigma_l}}{1+\sigma_l}\right)$$

Then in a symmetric equilibrium he household first-order conditions for external habit and internal habit respectively are

$$1 = \mathbb{E}_t\left[R_{t+1}\Lambda_{t,t+1}\right]$$
$$\Lambda_{t,t+1} = \beta_{g,t+1}\frac{\lambda_{t+1}}{\lambda_t}$$
$$\beta_{g,t} = \beta\,(1+g_t)^{-\sigma_c}$$
$$U_{H,t} = -H_t^{\sigma_l}(C_t - \chi C_{t-1}/(1+g_t))^{-\sigma_c} \exp\left(\frac{(\sigma_c-1)H_t^{1+\sigma_l}}{1+\sigma_l}\right)$$
$$\frac{U_{H,t}}{\lambda_t} = -W_t$$

where for external habit and internal habit respectively we have

$$\lambda_t = U_{C,t}$$
$$\lambda_t = U_{C,t} - \beta\chi\mathbb{E}_t[U_{C,t+1}]$$

Parameter σ_l is referred to by Smets and Wouters (2007) as the labour supply elasticity.

A.2 Sticky Prices

First we introduce a retail sector producing differentiated goods under monopolistic competition. This sector converts homogeneous output from a competitive wholesale sector. The aggregate prices in the two sectors are given by P_t and P_t^W respectively and $P_t > P_t^W$ from the *mark-up* possible under monopolistic competition. The *real marginal cost* of producing each differentiated good $MC_t \equiv \frac{P_t^W}{P_t}$. In the RBC model $P_t = P_t^W$ so $MC_t = 1$ and the *marginal cost is constant*. In the NK model retailers are locked into price-contracts and cannot their prices every period. Their marginal costs therefore vary. In periods of high demand they simply increase output until they are able to change prices.

The retail sectors then uses a homogeneous wholesale good to produce a basket of differentiated goods for consumption

$$C_t = \left(\int_0^1 C_t(m)^{(\zeta-1)/\zeta} dm \right)^{\zeta/(\zeta-1)} \tag{A.1}$$

where $\zeta > 1$ is the elasticity of substitution. For each m, the consumer chooses $C_t(m)$ at a price $P_t(m)$ to maximize (A.1) given total expenditure $\int_0^1 P_t(m)C_t(m)dm$. This results in a set of consumption demand equations for each differentiated good m with price $P_t(m)$ of the form

$$C_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta} C_t$$

where $P_t = \left[\int_0^1 P_t(m)^{1-\zeta} dm \right]^{\frac{1}{1-\zeta}}$. P_t is the aggregate price index. Note that C_t and P_t are Dixit-Stiglitz aggregators – see Dixit and Stiglitz (1977). Demand for investment and government services takes the same form, so in aggregate

$$Y_t(m) = \left(\frac{P_t(m)}{P_t} \right)^{-\zeta} Y_t$$

Following Calvo (1983), we now assume that there is a probability of $1 - \xi_p$ at each period that the price of each retail good m is set optimally to $P_t^0(m)$. If the price is not re-optimized, then it is held fixed.[13] For each retail producer m, given its real marginal cost $MC_t = \frac{P_t^W}{P_t}$, the objective is at time t to choose $\{P_t^0(m)\}$ to maximize discounted real profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_t^0(m) - P_{t+k} MC_{t+k} \right]$$

subject to

$$Y_{t+k}(m) = \left(\frac{P_t^O(m)}{P_{t+k}} \right)^{-\zeta} Y_{t+k} \tag{A.2}$$

[13]Thus we can interpret $\frac{1}{1-\xi_p}$ as the average duration for which prices are left unchanged.

where $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the (non-stationarized) stochastic discount factor[14] over the interval $[t, t+k]$. The solution to this optimization problem is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P_t^0(m) - \frac{1}{(1-1/\zeta)} P_{t+k} MC_{t+k} \right] = 0$$

Using (A.2) and rearranging this leads to

$$P_t^O = \frac{1}{(1-1/\zeta)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} (P_{t+k})^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} (P_{t+k})^{\zeta} Y_{t+k}} \tag{A.3}$$

where the m index is dropped as all firms face the same marginal cost so the right-hand side of the equation is independent of firm size or price history.

By the law of large numbers the evolution of the price index is given by

$$P_t^{1-\zeta} = \xi_p P_{t-1}^{1-\zeta} + (1-\xi_p)(P_t^0)^{1-\zeta} \tag{A.4}$$

Now define k periods ahead inflation as

$$\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t}$$

To ease the notation in what follows we denote $\Pi_t = \Pi_{t-1,t}$ and $\Pi_{t+1} = \Pi_{t,t+1}$.

We can now write the fraction (A.3)

$$\frac{P_t^O}{P_t} = \frac{1}{(1-1/\zeta)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} (\Pi_{t,t+k})^{\zeta-1} Y_{t+k}}$$

and (A.4) as

$$1 = \xi_p (\Pi_t)^{\zeta-1} + (1-\xi_p) \left(\frac{P_t^O}{P_t} \right)^{1-\zeta}$$

A.3 Price Dynamics

In order to set up the model in non-linear form as a set of difference equations, required for software packages such a Dynare, we need to represent the price dynamics as *difference equations*. .

First we assume a zero-growth steady state so that we do not yet need to stationarize

[14] We stationarize the model later.

any variables. Then using the Lemma in that section , price dynamics are given by

$$
\begin{aligned}
\frac{P_t^0}{P_t} &= \frac{J_t^p}{JJ_t^p} \\
JJ_t^p - \xi_p \mathbb{E}_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta-1} JJ_{t+1}^p] &= Y_t \\
J_t^p - \xi_p \mathbb{E}_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta} J_{t+1}^p] &= \left(\frac{1}{1-\frac{1}{\zeta}}\right) Y_t MC_t MCS_t \\
1 &= \xi_p \Pi_t^{\zeta-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{1-\zeta} \\
MC_t &= \frac{P_t^W}{P_t} = \frac{W_t}{F_{H,t}} \qquad \text{(A.5)}
\end{aligned}
$$

where (A.5) allows for $P_t \neq P_t^W$. We have also introduced a mark-up shock MCS_t to MC_t. Notice that the real marginal cost, MC_t, is no longer fixed as it was in the RBC model.

A.4 Indexing

Prices are now are indexed to last period's aggregate inflation, with a price indexation parameter γ_p. Then the price trajectory with no re-optimization is given by $P_t^O(j)$, $P_t^O(j)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p}$, $P_t^O(j)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_p}$, $\cdots$. where $Y_{t+k}(m)$ is given by (A.2) with indexing so that

$$
Y_{t+k}(m) = \left(\frac{P_t^O(m)}{P_{t+k}}\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p}\right)^{-\zeta} Y_{t+k}
$$

With indexing by an amount $\gamma_p \in [0,1]$ and an exogenous mark-up shock MS_t as before, the optimal price-setting first-order condition for a firm j setting a new optimized price $P_t^0(j)$ is now given by

$$
P_t^0 = \frac{\frac{\zeta}{\zeta-1}\mathbb{E}_t\left[\sum_{k=0}^{\infty}\xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} MC_{t+k} MS_{p,t+k} Y_{t+k}\right]}{\mathbb{E}_t\left[\sum_{k=0}^{\infty}\xi_p^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma}\right]}.
$$

Price dynamics are now given by

$$
\begin{aligned}
\frac{P_t^0}{P_t} &= \frac{J_t^p}{JJ_t^p} \\
JJ_t^p - \xi_p \mathbb{E}_t[\Lambda_{t,t+1}\tilde{\Pi}_{t+1}^{\zeta-1} JJ_{t+1}^p] &= Y_t \\
J_t^p - \xi_p \mathbb{E}_t[\Lambda_{t,t+1}\tilde{\Pi}_{t+1}^{\zeta} J_{t+1}^p] &= \frac{\zeta}{\zeta-1} MC_t MS_{p,t} Y_t \\
\tilde{\Pi}_t &\equiv \frac{\Pi_t}{\Pi_{t-1}^{\gamma_p}}
\end{aligned}
$$

$$1 = \xi_p \tilde{\Pi}_t^{\zeta-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{1-\zeta}$$

A.5 Price Dynamics in a Non-Zero-Growth Steady State

Stationarizing J_t^p and JJ_t^p as in the RBC model, price dynamics with indexing become

$$\begin{aligned}
\frac{P_t^0}{P_t} &= \frac{J_t^p}{JJ_t^p} \\
JJ_t^p - \xi_p \mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}_{t+1}^{\zeta-1} JJ_{t+1}^p] &= Y_t \\
J_t^p - \xi_p \mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}_{t+1}^{\zeta} J_{t+1}^p] &= \frac{\zeta}{\zeta-1} MC_t MS_{p,t} Y_t \\
\tilde{\Pi}_t &\equiv \frac{\Pi_t}{\Pi_{t-1}^{\gamma_p}} \\
1 &= \xi_p \tilde{\Pi}_t^{\zeta-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{1-\zeta}
\end{aligned}$$

A.6 Sticky Wages

To introduce wage stickiness we now assume that each household supplies homogeneous labour at a nominal wage rate $W_{h,t}$ to a monopolistic trade-union who differentiates the labour and sells type $H_t(j)$ at a nominal wage $W_{n,t}(j) > W_{h,t}$ to a labour packer in a sequence of Calvo staggered nominal wage contracts. The real wage is then defined as $W_t \equiv \frac{W_{n,t}}{P_t}$. We now have to distinguish between *price inflation* which now uses the notation $\Pi_t^p \equiv \frac{P_t}{P_{t-1}}$ and *wage inflation*, $\Pi_t^w \equiv \frac{W_{n,t}}{W_{n,t-1}} = \frac{W_t \Pi_t^p}{W_{t-1}}$.

As with price contracts we employ Dixit-Stiglitz quantity and price aggregators. Calvo probabilities are now ξ_p and ξ_w for price and wage contracts respectively. The competitive labour packer forms a composite labour service according to $H_t = \left(\int_0^1 H_t(j)^{(\mu-1)/\mu} dj\right)^{\mu/(\mu-1)}$ and sells onto the intermediate firm. where $\mu > 1$ is the elasticity of substitution. For each j, the labour packer chooses $H_t(j)$ at a wage $W_{n,t}(j)$ to maximize H_t given total expenditure $\int_0^1 W_{n,t}(j) H_t(j) dj$. This results in a set of labour demand equations for each differentiated labour type j with wage $W_{n,t}(j)$ of the form

$$H_t(j) = \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\mu} H_t \tag{A.6}$$

where $W_{n,t} = \left[\int_0^1 W_{n,t}(j)^{1-\mu} dj\right]^{\frac{1}{1-\mu}}$ is the aggregate nominal wage index. H_t and $W_{n,t}$ are Dixit-Stigliz aggregators for the labour market.

Wage setting by the trade-union again follows the standard Calvo framework supplemented with indexation. At each period there is a probability $1-\xi_w$ that the wage is set optimally. The optimal wage derives from maximizing discounted profits. For those trade-unions unable to reset, wages are indexed to last period's aggregate inflation, with wage indexation parameter γ_w. Then as for price contracts the wage rate trajectory with no re-optimization is given by $W^O_{n,t}(j)$, $W^O_{n,t}(j)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_w}$, $W^O_{n,t}(j)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_w}$, $\cdots$. The trade union then buys homogeneous labour at a nominal price $W_{h,t}$ and converts it into a differentiated labour service of type j. The trade union time t then chooses $W^O_{n,t}(j)$ to maximize real profits

$$\mathbb{E}_t\sum_{k=0}^{\infty}\xi_w^k\frac{\Lambda_{t,t+k}}{P_{t+k}}H_{t+k}(j)\left[W^O_{n,t}(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w}-W_{h,t+k}\right]$$

where using (A.6) with indexing $H_{t+k}(j)$ is given by

$$H_{t+k}(j)=\left(\frac{W^O_{n,t}(j)}{W_{n,t+k}}\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w}\right)^{-\mu}H_{t+k}$$

and μ is the elasticity of substitution across labour varieties.

This leads to the following first-order condition

$$\mathbb{E}_t\sum_{k=0}^{\infty}\xi_w^k\frac{\Lambda_{t,t+k}}{P_{t+k}}H_{t+k}(j)\Big[W^0_t(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w}-\frac{\mu}{\mu-1}W_{h,t+k}\Big]=0$$

and hence by analogy with price-setting, this leads to the optimal real wage

$$\frac{W^O_{n,t}}{P_t}=\frac{\mu}{\mu-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_w^k\Lambda_{t,t+k}\left(\Pi^w_{t,t+k}\right)^{\zeta}H_{t+k}\frac{W_{h,t+k}}{P_{t+k}}}{\mathbb{E}_t\sum_{k=0}^{\infty}\xi_w^k\Lambda_{t,t+k}\left(\Pi^w_{t,t+k}\right)^{\zeta}\left(\Pi^p_{t,t+k}\right)^{-1}H_{t+k}}=\frac{J^w_t}{JJ^w_t}$$

Then by the law of large numbers the evolution of the wage index is given by

$$W^{1-\mu}_{n,t}=\xi_w\left(W_{n,t-1}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_w}\right)^{1-\mu}+(1-\xi_w)(W^0_{n,t}(j))^{1-\mu}$$

A.7 Price and Wage Dynamics

We now apply the analysis of A.3-A.5 to wage dynamics and bring the two forms together. The model is now stationarized.

$$\begin{aligned}
\Pi^p_t &\equiv \frac{P_t}{P_{t-1}}\\
\tilde{\Pi}^p_t(\gamma) &\equiv \frac{\Pi^p_t}{\Pi^{\gamma}_{p,t-1}}\\
JJ^p_t-\xi_p\mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}^p_{t+1}(\gamma_p)^{\zeta-1}JJ^p_{t+1}] &= Y_t
\end{aligned}$$

$$
\begin{aligned}
J_t^p - \xi_p \mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}_{t+1}^p(\gamma_p)^\zeta J_{t+1}^p] &= \frac{\zeta}{\zeta-1} Y_t MC_t MS_{p,t} \\
1 &= \xi_p \tilde{\Pi}_t^p(\gamma_p)^{\zeta-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{1-\zeta} \\
\frac{P_t^O}{P_t} &= \frac{J_t^p}{JJ_t^p}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_t^w &\equiv \frac{W_{n,t}}{W_{n,t-1}} = (1+g_t)\frac{\Pi_t W_t}{W_{t-1}} && \text{(A.7)} \\
\tilde{\Pi}_t^w &\equiv \frac{\Pi_t^w}{(\Pi_{t-1}^p)^{\gamma_w}} && \text{(A.8)} \\
MRS_t &= -\frac{U_{H,t}}{U_{C,t}} = \frac{W_{h,t}}{P_t} && \text{(A.9)} \\
JJ_t^w - \xi_w \mathbb{E}_t\left[\Lambda_{t,t+1}\frac{(\tilde{\Pi}_{t,t+1}^w)^\mu}{\tilde{\Pi}_{t,t+1}^p(\gamma_w)} JJ_{t+1}^w\right] &= H_{d,t} && \text{(A.10)} \\
J_t^w - \xi_w \mathbb{E}_t\left[(1+g_{t+1})\Lambda_{t,t+1}\tilde{\Pi}_{w,t+1}^\mu J_{t+1}^w\right] &= -\frac{\mu}{\mu-1} MRS_t MS_{w,t} H_{d,t} && \text{(A.11)}
\end{aligned}
$$

$$
\begin{aligned}
(W_{n,t})^{1-\mu} &= \xi_w\left((W_{n,t-1})\frac{1}{\tilde{\Pi}_t^p(\gamma_w)}\right)^{1-\mu} + (1-\xi_w)\left(W_{n,t}^O\right)^{1-\mu} \Rightarrow \\
1 &= \xi_w\left(\frac{\Pi_t^w \tilde{\Pi}_{p,t}(\gamma_w)}{\Pi_t^p}\right)^{\mu-1} + (1-\xi_w)\left(\frac{W_{n,t}^O(j)}{W_{n,t}}\right)^{1-\mu} && \text{(A.12)} \\
W_t^O &\equiv \frac{W_{n,t}^O}{W_{n,t}} = \frac{W_{n,t}^O/P_t}{W_{n,t}/P_t} = \frac{J_t^w}{W_t JJ_t^w} && \text{(A.13)} \\
\Pi_t^w &= (1+g_t)\frac{\Pi_t W_t}{W_{t-1}} && \text{(A.14)}
\end{aligned}
$$

A.8 Capacity Utilization and Fixed Costs of Production

We now add two remaining features to the model. As in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) we assume that using the stock of capital with intensity u_t produces a cost of $a(u_t)K_t$ units of the composite final good. The functional form is chosen consistent with the literature:

$$
a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2 \qquad \text{(A.15)}
$$

and satisfies $a(1) = 0$ and $a'(1), a''(1) > 0$. Then we must add a term $(r_t^K - a(u_t)K_t$ to the household budget constraint on the income side where r_t^K is the rental rate leading to the following first-order condition determines capacity utilization:

$$r_t^K = a'(u_t) \tag{A.16}$$

Capital now enters the production function as $u_t K_{t-1}$.

The final change is to add fixed costs F, necessary to transform homogeneous wholesale goods into differentiated retail goods. To pin down F we make the assumption that entry occurs until retail profits are eliminated in the steady state , i.e., $P^W Y^W = PY$. It follows that

$$\frac{P^W}{P} = MC = \frac{Y}{Y^W} = \frac{\left(1 - \frac{F}{Y^W}\right)}{\Delta_p} \tag{A.17}$$

It follow that

$$\frac{F}{Y^W} = 1 - \Delta_p MC \tag{A.18}$$

For the zero inflation, $MC = 1 - \frac{1}{\zeta}$ and $\Delta_p = \Delta_w = 1$ and therefore $\frac{F}{Y^W} = \frac{1}{\zeta}$.

A.9 Price and Wage Dispersion

The output and labour market clearing conditions must take into account relative price dispersion across varieties and wage dispersion across firms. Integrating across all firms, taking into account that the capital-labour ratio is common across firms and that the wholesale sector is separated from the retail sector we obtain aggregate demand for intermediate (wholesale) goods necessary to produce final retail goods as

$$Y_t^W - F = \int_0^1 \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} dm(C_t + I_t + G_t) = \Delta_t^p Y_t$$

where labour market clearing gives total demand for labour, H_t^d, as

$$H_t = \int_0^1 H_t(j)dj = \int_0^1 \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\mu} dj\, H_t^d = \Delta_t^w H_t^d$$

where the price dispersion is given by $\Delta_t^p = \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\zeta} df$ and wage dispersion is given by $\Delta_t^w = \int_0^1 \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\mu} dj$. We have:

$$\Delta_t^p = \xi_p + \tilde{\Pi}_t^{\zeta} \Delta_{t-1}^p + (1 - \xi_p) \left(\frac{P_t^O}{P_t}\right)^{-\zeta}$$

$$\Delta_t^w = \xi_w \tilde{\Pi}_{w,t}^{\mu} \Delta_{t-1}^w + (1 - \xi_w) \left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{-\mu}$$

A.10 Summary of Supply Side

Wholesale, Retail and capital producer firm behaviour is given by

$$
\begin{aligned}
\textbf{Wholesale Production} &: \; Y_t^W = (A_t H_t^d)^\alpha K_{t-1}^{1-\alpha} \\
\textbf{Retail Aggregate Production} &: \; Y_t = \frac{Y_t^W - F}{\Delta_t^p} \\
\textbf{Aggregate Employed Labour} &: \; H_t^d = \frac{H_t}{\Delta_t^w} \\
\textbf{Labour Demand} &: \; W_t = \frac{P_t^W}{P_t} F_{H,t} = \frac{P_t^W}{P_t} \frac{\alpha Y_t^W}{H_t^d} \\
\textbf{Capital Demand} &: \; r_t^K = \frac{P_t^W}{P_t} F_{K,t} = \frac{P_t^W}{P_t} \frac{(1-\alpha) Y_t^W}{K_{t-1}}
\end{aligned}
$$

where K_t is *end-of-period* $[t, t+1]$ capital, W_t is the wage rate of the composite differentiated labour orovided by the labour packer (trade-union) and Δ_t^p and Δ_t^w are price dispersion and wage dispersion (defined below), r_t^K is the rental net rate for capital and we have imposed labour demand equal to labour supply in a labour market equilibrium. Production is assumed to be Cobb-Douglas.

Capital accumulation with investment adjustment costs carried out by capital goods producers is given by

$$
\begin{aligned}
K_t &= (1-\delta)K_{t-1} + (1 - S(X_t)) I_t IS_t \\
X_t &\equiv \frac{I_t}{I_{t-1}} \\
S(X_t) &= \phi_X (X_t - 1 - g)^2 \\
S'(X_t) &= 2\phi_X (X_t - 1) \\
Q_t IS_t (1 - S(X_t) - X_t S'(X_t)) &+ \mathbb{E}_t \left[\Lambda_{t,t+1} Q_{t+1} IS_{t+1} S'(X_{t+1}) X_{t+1}^2 \right] = 1
\end{aligned}
$$

where I_t, and Q_t are investment and the real price of capital respectively. IS_t is a capital specific shock process. $S(X_t)$ are investment adjustment costs equal to zero in a balance growth steady state with output, consumption, capital, investment and the real wage growing at a rate g.

Then this completes the supply side with price and wage dynamics and dispersion as given in sections A.7 and A.9.

A.11 Capital Return and Expected Spread

The gross return on capital by

$$R_t^K = \left[\frac{r_t^K + (1-\delta)Q_t}{Q_{t-1}} \right]$$

Then in the *absence of financial frictions* including the risk-premium shock RPS_t we have *arbitrage* between discounted returns on capital and deposits given by

$$\mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}^K] = \mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}] = 1$$

In the main model, where we include financial frictions, we have

$$\mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}^K] \neq \mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}] = 1$$

A.12 The Monetary Rule and Output Equilibrium

The nominal interest rate is given by one of the following Taylor-type rules

$$\begin{aligned} \log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\Big[\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) \\ &+ \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\Big] + \epsilon_{MPS,t} \end{aligned}$$

where $\epsilon_{M,t}$ is a monetary policy shock process. θ_π and θ_y are the long-run elasticities of the inflation and output respectively with respect to the interest rate. The "Taylor principle" requires $\theta_\pi > 1$. The conventional Taylor rule stabilizes output about its flexi-price level which is that found by solving the RBC core of this model or simply allowing the contract parameter ξ_p to tend to zero. Unlike the implementable form, this requires observations of the output gap $\frac{Y_t}{Y_t^F}$ to implement and monitor.[15] The output equilibrium is given by

$$Y_t = C_t + G_t + I_t$$

A.13 Full NK Model Listing

The full model in stationarized form is given by:

A.14 Dynamic Model

$$g = \bar{g}$$

[15] Technically this should pose no problems in a perfect information rational expectations equilibrium, but the rationale for 'simple rules' is to have policies that are easy to observe without relying on the perfect information solution.

$$
\begin{aligned}
\beta_{g,t} &= \beta\ (1+g_t)^{-\sigma_c} \\
U_t &= \frac{(C_t - \chi C_{t-1}/(1+g_t))^{1-\sigma_c} \exp\left(\frac{(\sigma_c-1)(H_t)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c} \\
CE_t &= \frac{(1.01(C_t - \chi C_{t-1}/(1+g_t)))^{1-\sigma_c} \exp\left(\frac{(\sigma_c-1)(H_t)^{1+\sigma_l}}{1+\sigma_l}\right) - 1}{1-\sigma_c} - U_t \\
&+ \mathbb{E}_t[(1+g_{t+1})\ \beta_{g,t+1}\, CE_{t+1}] \\
\Omega_t &= U_t + \beta\, \mathbb{E}_t[(1+g)^{1-\sigma_c}\ exp\left((1-\sigma_c)\ \epsilon_{Atrend,t+1}\right)\ \Omega_{t+1}] \\
U_{C_t} &= (C_t - \chi C_{t-1}/(1+g_t))^{-\sigma_c} \exp\left(\frac{(\sigma_c-1)H_t^{1+\sigma_l}}{1+\sigma_l}\right) \\
U_{H_t} &= -H_t^{\sigma_l}(C_t - \chi C_{t-1}/(1+g_t))^{-\sigma_c} \exp\left(\frac{(\sigma_c-1)H_t^{1+\sigma_l}}{1+\sigma_l}\right) \\
\lambda_t &= \mathbb{E}_t[\beta_{g,t+1}\, R_{t+1}]\, RPS_t\, \lambda_{t+1} \\
\lambda_t &= U_{C,t} - \chi\, \mathbb{E}_t[\beta_{g,t+1} U_{C,t+1}] \\
\frac{-U_{H_t}}{\lambda_t} &= W_{h,t} \\
R_t &= \frac{R_{n,t-1}}{\Pi_t} \\
Y_t &= \frac{Y_t^W - F}{\Delta_t^p} \\
H_{d,t} &= \frac{H_t}{\Delta_t^w} \\
Y_t^W &= (H_{d,t}\, A_t)^\alpha \left(\frac{K_{t-1}}{1+g_t}\right)^{1-\alpha} \\
R_t^K &= \frac{\left(\frac{Y_t^W\ (1-\alpha)\ MC_t}{\frac{K_{t-1}}{1+g_t}} + (1-\delta)\ Q_t\right)}{Q_{t-1}} \\
\Lambda_{t-1,t} &= \frac{\beta_{g,t}\, \lambda_t}{\lambda_{t-1}} \\
1 &= Q_t\ \left(1 - S_t - X_t\, S_t'\right) + \mathbb{E}_t[\Lambda_{t,t+1}\, Q_{t+1}\, S_{t+1}'\ (X_{t+1})^2] \\
\frac{\alpha\, MC_t\, Y_t^W}{H_t} &= W_t \\
MC_t &= \frac{P_t^W}{P_t} \\
K_t &= \left((1-S_t)\ I_t + \frac{K_{t-1}\ (1-\delta)}{1+g_t}\right) \\
X_t &= \frac{(1+g_t)\ I_t}{I_{t-1}} \\
S_t &= \phi_X\ (X_t - 1 - g)^2 \\
S_t' &= 2\, \phi_X\ (X_t - 1 - g)
\end{aligned}
$$

$$
\begin{aligned}
1 &= \Lambda_{t,t+1}\, R^K_{t+1} = 1 \\
Y_t &= C_t + I_t + G_t \\
Y_t &= JJ^p_t - \mathbb{E}_t[(1+g_{t+1})\ \Lambda_{t,t+1}\, \tilde{\Pi}^{\zeta-1}_{t+1}\, JJ^p_{t+1}] \\
\frac{\zeta}{\zeta-1}\, Y_t\, MC_t\, MCS_t &= J^p_t - \mathbb{E}_t\left[(1+g_{t+1})\ \Lambda_{t,t+1}\, \tilde{\Pi}^{\zeta}_{t+1}\, J^p_{t+1}\right] \\
\Lambda_{t,t+1} &= \frac{\beta_{g,t+1} U_{C,t+1}}{U_{C,t}} \\
\tilde{\Pi}_t &= \frac{\Pi_t}{\Pi^{\gamma_p}_{t-1}} \\
P^O_t &= \frac{J^p_t}{JJ^p_t} \\
1 &= \tilde{\Pi}^{\zeta-1}_t + (1-\xi_p)\ (P^O_t)^{1-\zeta} \\
\Delta^p_t &= \xi_p\, \tilde{\Pi}^{\zeta}_t\, \Delta^p_{t-1} + (1-\xi_p)\ (P^O_t)^{(-\zeta)} \\
\Pi^w_t &= \Pi_t \frac{W_t(1+g_t)}{W_{t-1}} \\
\tilde{\Pi}^w_t &= \frac{\Pi_t}{\Pi^{\gamma_w}_{t-1}} \\
H_t &= JJ^w_t - \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}\, \xi_w\, (\tilde{\Pi}^w_{t+1})^{\mu_w}}{\tilde{\Pi}_{t+1}(\gamma_w)}\, JJ^w_{t+1}\right] \\
\frac{\mu_w}{\mu_w-1}\, W_{h,t}\, H_t\, MRSS_t &= J^w_t - \mathbb{E}_t[(1+g_{t+1})\Lambda_{t,t+1}\, \xi_w\, (\tilde{\Pi}^w_{t+1})^{\mu_w}\, J^w_{t+1}] \\
W^O_t &= \frac{J^w_t}{W_t\, JJ^w_t} \\
1 &= \xi_w \left(\frac{\Pi^w_t\, \tilde{\Pi}_t(\gamma_w)}{\Pi_t}\right)^{\mu_w-1} + (1-\xi_w)\ (W^O_t)^{1-\mu_w} \\
\Delta^w_t &= \xi_w\, (\tilde{\Pi}^w_t)^{\mu_w}\, \Delta^w_{t-1} + (1-\xi_w)\ (W^O_t)^{-\mu_w} \\
Invmarkup_t &= \frac{W_{h,t}}{W_t} \\
\log\left(\frac{R_{n,t}}{\bar{R}n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{\bar{R}n}\right) + (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\bar{\Pi}}\right)\right. \\
&+\ \left.\theta_y \log\left(\frac{Y_t}{\bar{Y}}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \log(MPS_t)
\end{aligned}
$$

with AR(1) processes for A_t, G_t, MC_t, $MRSS_t$, IS_t, MPS_t and RPS_t.

A.15 Balanced Growth Steady State

With non-zero steady state growth, the steady state for the rest of the system is the same as the zero-growth RBC model except for the following relationships: for particular steady state inflation rate $\Pi_p = \Pi_w > 1$ the NK features of the blanced growth steady state become:

$$
\begin{aligned}
R_n &= \Pi R \\
\tilde{\Pi}_p(\gamma) &\equiv \Pi^{1-\gamma} \\
\frac{P^O}{P} = \frac{J^p}{JJ^p} &= \left(\frac{1-\xi_p\tilde{\Pi}_p(\gamma_p)^{\zeta-1}}{1-\xi_p}\right)^{\frac{1}{1-\zeta}} \\
MC = \frac{P^W}{P} &= \left(1-\frac{1}{\zeta}\right)\frac{J^p(1-\beta(1+g)\xi_p\tilde{\Pi}_p(\gamma_p)^{\zeta})}{H_p(1-\beta(1+g)\xi_p\tilde{\Pi}_p(\gamma_p)^{\zeta-1})} \\
&= \text{Inverse of price mark-up} \\
\Delta_p &= \frac{1-\xi_p}{1-\xi_p\tilde{\Pi}_p(\gamma_p)^{\zeta}}\left(\frac{J^p}{JJ^p}\right)^{-\zeta}
\end{aligned}
$$

and for wage dynamics

$$
\begin{aligned}
\frac{W^O}{W} = \frac{\frac{J^w}{JJ^w}}{\frac{W}{P}} &= \left(\frac{1-\xi_w\tilde{\Pi}_p(\gamma_w)^{\mu-1}}{1-\xi_w}\right)^{\frac{1}{1-\mu}} \\
\frac{J^w}{JJ^w} &= MS_w\frac{W_h}{P}\frac{(1-\beta\xi_w(1+g)\tilde{\Pi}_p(\gamma_w)^{\mu-1}}{(1-\beta\xi_w\tilde{\Pi}_p(\gamma_w)^{\mu})} \\
\text{i.e., } \frac{\frac{W_h}{P}}{\frac{W}{P}} &= \left(1-\frac{1}{\mu}\right)\frac{\frac{J^w}{JJ^w}}{\frac{W}{P}}\frac{(1-\beta\xi_w\tilde{\Pi}_p(\gamma_w)^{\mu}}{(1-\beta\xi_w(1+g)\tilde{\Pi}_p(\gamma_w)^{\mu-1})} \\
&= \text{Inverse of wage mark-up} \\
\Delta_w &= \frac{1-\xi_w}{1-\xi_w\tilde{\Pi}_p(\gamma_w)^{\mu}}\left(\frac{\frac{J^w}{JJ^w}}{\frac{W}{P}}\right)^{-\mu}
\end{aligned}
$$

B The Banker's Problem Solution

The solution is assumed to take the form

$$V_t = \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}n_{B,t+1}] \tag{B.19}$$

We write the Bellman equation as

$$
\begin{aligned}
V_{t-1} &= \max_{l_t,m_t}\mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma_B)n_t+\sigma_B V_t] \\
&= \max_{l_t,m_t}\mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma_B)n_t+\sigma_B\mathbb{E}_t(\Lambda_{t,t+1}n_{t+1})]
\end{aligned} \tag{B.20}
$$

where corresponding to (15)

$$\mathbb{E}_t(\Lambda_{t,t+1}n_{t+1}) = \mathbb{E}_t[\Lambda_{t,t+1}(R_{t+1}n_t+(R^B_{t+1}-R_{t+1})l_t-(R^M_{t+1}-R_{t+1})m_t)]$$

This is subject to the condition that $V_t \geqslant \theta[l_t - \omega m_t]$, which implies the constraint

$$\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}[R_{t+1}n_{B,t} + (R^B_{t+1} - R_{t+1})l_t - (R^M_{t+1} - R_{t+1})m_t)] \geqslant \theta(l_t - \omega m_t) \qquad \text{(B.21)}$$

If $\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}[(R^B_{t+1} - R_{t+1})l_t + (R^M_{t+1} - R_{t+1})m_t)] < \theta(l_t - \omega m_t)$, then maximization takes place if and only if the constraint binds, so that the solution is:

$$l_t = \frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]}n_{B,t} + m_t\frac{(\theta\omega - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^M_{t+1} - R_{t+1})]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]}.$$
(B.22)

The arbitrage condition between the interest rates implies the following relation:

$$\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^M_{t+1} - R_{t+1})] = \omega\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]. \qquad \text{(B.23)}$$

Substituting this to B.22 it simplifies to:

$$l_t = \frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]}n_{B,t} + \omega m_t. \qquad \text{(B.24)}$$

Substituting B.24 to the terminal wealth:

$$V_t \;=\; \mathbb{E}_t n_t[(\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})(\frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]})) + \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_t)]$$

and the Bellman equation becomes

$$\begin{aligned} V_{t-1} &= \max_{l_t,m_t} \mathbb{E}_{t-1}\Lambda_{t-1,t}[(1-\sigma_B)n_t + \sigma_B V_t] \\ &= (1-\sigma_B)n_{B,t} + \sigma_B n_{B,t}\{\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})\left(\frac{\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}}{\theta - \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})}\right) \\ &+ \mathbb{E}_t(\Lambda_{t,t+1}\Omega_{t+1}R_t)\} \end{aligned}$$

It follows that

$$\Omega_t = (1-\sigma_B)+\sigma_B[(\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1}-R_{t+1})\left(\frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})]})\right)+\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}R_t)]$$
(B.25)

Equivalently, defining for banks the maximum adjusted leverage

$$\phi^B_t = (l_t - \omega m_t)/n_{B,t} \qquad \text{(B.26)}$$

we can rewrite this last equation as

$$\Omega_t = 1 - \sigma_B + \sigma_B\theta\phi^B_t. \qquad \text{(B.27)}$$

C Financial Frictions Model Listing

In this section we describe the system of equations of the financial frictions part of the model. The stationarized form can be summarized as:

$$
\begin{aligned}
\mathbb{E}_t[R^K_{t+1}] &= \mathbb{E}_t[\rho(\bar{\psi}_{t+1})R^B_{t+1}] \\
(1+g_t)N_{E,t} &= (\sigma_E + \xi_E)(1-\Gamma(\bar{\psi}_t))R^K_t Q_{t-1}K_{t-1} \\
\phi_t &= \frac{(\phi_t - 1)\mathbb{E}_t\left[R^B_{t+1}\right]}{\mathbb{E}_t\left[R^K_{t+1}\left[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})\right]\right]} \\
\phi_t &= \frac{Q_t K_t}{N_{E,t}} \\
\rho(\bar{\psi}_{t+1}) &= \frac{\Gamma'(\bar{\psi}_{t+1})}{\left[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1-\Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))\right]} \\
L_t &= Q_t K_t - N_{E,t} \\
\bar{\psi}_t &= \frac{R_{l,t-1}L_{t-1}}{R^K_t Q_{t-1}K_{t-1}}\frac{1}{\Pi_t} \\
R^K_t &= \frac{r^K_t u_t - \alpha(u_t) + (1-\delta)Q_t}{Q_{t-1}} \\
r^K_t &= \frac{(1-\alpha)P^W_t Y^W_t}{u_{t-1}K_{t-1}/(1+g_t)} \\
L_t &= \phi^B_t N_{B,t} + \omega M_t \\
\phi^B_t &= \frac{\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}]}{\theta - \Omega_{t+1}\mathbb{E}_t\Lambda_{t,t+1}[R^B_{t+1} - R_{t+1}]} \\
\Omega_t &= 1 - \sigma_B + \sigma_B\theta\phi^B_t \\
N_{B,t}(1+g_t) &= (\sigma_B + \xi_B)R^B_t L_{t-1} - \sigma_B(R_t D_{t-1} + R^M_t M_{t-1}) \\
D_t &= L_t - N_{B,t} - M_t \\
\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^M_{t+1} - R_{t+1})] &= \omega\mathbb{E}_t[\Lambda_{t,t+1}\Omega_{t+1}(R^B_{t+1} - R_{t+1})] \\
lever_t &= \frac{L_t}{N_{B,t} + M_t} \\
T_t &= G_t + M_t - R_{m,t}M_{t-1} \\
M_t &= \chi_{m,t}L_t \\
Y_t &= C_t + C_{E,t} + G_t + I_t + \mu G(\bar{\psi}_t))R^K_t Q_{t-1}K_{t-1}/(1+g_t) + \alpha(u_t)K_{t-1}/(1+g_t \\
(1+g_t)C_{E,t} &= (1-\sigma_E)(1-\xi_E)(1-\Gamma(\bar{\psi}_t))R^K_t Q_{t-1}K_{t-1}
\end{aligned}
$$

D Steady State Derivations

D.1 Steady State of the Bankers Problem

I begin by finding the steady state of the financial sector variables and then I proceed with the real sector variables. A method that simplifies the calculations is to divide all variables in the bankers' problem over the loans (L) and in the entrepreneurs' problem over the capital K. Firstly, I show the steady state values for Q, R, Λ. From the capital producers problem we have that $Q = 1$ and from the Euler equation, we have that $R = \frac{1}{\beta^g}$ and $\Lambda = \beta^g$.

The goal here is to have two equations with unknowns the bank leverage (ϕ^B) and the interest rate on loans (R^B). The incentive constraint of the bank in steady state is

$$L = \phi^B N^B + \omega M, \tag{D.28}$$

where $M = \chi_m L \rightarrow \frac{M}{L} = \chi_m$ By dividing (D.28) over loans we have $\frac{L}{L} = \phi^B \frac{(N^B)}{L} + \omega \frac{M}{L}$. Rearranging terms :

$$\frac{N^B}{L} = \frac{1}{\phi^B}(1 - \omega\chi_m).$$

From the bank's balance sheet constraint we have $D = L - N - M$. Dividing over L:

$$\frac{N^B}{L} = 1 - \frac{D}{L} - \chi_m. \tag{D.29}$$

The bank's net worth is $N^B(1+g) = (\sigma^B + \xi^B)(R^B L) - \sigma^B(RD + R^M M)$. Again dividing over L and rearranging terms, yields:

$$\frac{N^B}{L} = \frac{1}{1+g}[(\sigma^B + \xi^B)R^B - \sigma^B(R\frac{D}{L} + R^M\chi_m]. \tag{D.30}$$

Substituting (D.29) in (D.30) and using $R = 1/\beta^g$ we have

$$\frac{N^B}{L} = (\sigma^B + \xi^B)(R^B) - \sigma^B(\frac{1}{\beta^g}(1 - \frac{N}{L} - \chi_m) + R^M M)$$

Rearranging terms and substituting $R^M = \omega R^B + (1-\omega)R$

$$\frac{N^B}{L} = \frac{(\sigma^B + \xi^B)R^B - \sigma^B/\beta^g + \omega\sigma^B\chi_m(R - R^B)}{1 + g - \sigma^B/\beta^g} = \frac{1}{\phi^B}(1 - \omega\chi_m).$$

So we get the **first equation** for the steady state leverage,

$$\boxed{\phi^B = \frac{(1 + g - \sigma^B/\beta)(1 - \omega\chi_m)}{(\sigma^B + \xi)R_B - \sigma^B/\beta + \omega\sigma^B\chi_m(R - R^B)}} \tag{D.31}$$

Now I turn in finding the steady state value of the leverage using the definition of leverage. We know that

$$\phi^B = \frac{\nu_{d,j}}{\theta - spread}$$

We also know that $\nu_d = \Lambda\Omega R = \beta\Omega\frac{1}{\beta} = \Omega$ After substituting ν_d, the leverage (ϕ^B) becomes

$$\phi^B = \frac{\Omega}{\theta - \Lambda\Omega(R^B - R)}$$

. Rearranging terms and substituting Ω given by

$$\Omega = (1 - \sigma^B) + \sigma^B\phi^B\theta$$

the leverage yields:

$$\boxed{\phi^B = \frac{(1 - \sigma^B) + \sigma^B\phi^B\theta}{\theta - ((1 - \sigma^B) + \sigma^B\phi^B\theta)(\beta R^B - 1)}} \tag{D.32}$$

being **the second equation** in the system. Hence, we have 2 equations (D.31, D.32) and 2 unknowns (ϕ^B, R^B). After solving this system it is straightforward to find ($\frac{N}{L}, \frac{D}{L}$).

D.2 Steady State of the Entrepreneurs' Problem

Here the solution strategy is the same with the bankers' problem. We find two equations with only unknowns the entrepreneurial leverage (ϕ^E) and the return on capital (R^K).

$$\phi^E = \frac{QK}{N^E}$$

Rearranging,

$$\frac{N^E}{K} = \frac{1}{\phi^E}$$

From, the entrepreneur's net worth equation, $N^E(1 + g) = (\sigma^E + \xi^E)(1 - \Gamma(\bar{\psi}))QKR^k$ we get the entrepreneurial net worth in steady state. Dividing with capital and substituting $Q = 1$ we have:

$$\frac{N_E}{K} = \frac{1}{(1 + g)}(\sigma^E + \xi^E)(1 - \Gamma(\bar{\psi}))R^k.$$

From entrepreneurs balance sheet constraint $L = QK - N^E$, dividing with $K^s s$ we have

$$\frac{N_E}{K} = 1 - \frac{L}{K} = \frac{1}{\phi^E}$$

Hence and using the fact that $R^K = \rho(\psi)R^B$,

$$\boxed{\phi_E = \frac{1+g}{(\sigma_E + \xi_E)(1 - \Gamma(\bar{\psi}))\rho(\psi)R^B}} \tag{D.33}$$

which is **the first equation** for the system.

From the Zero Profit Condition , solving for ϕ_E we get

$$\phi_E = -\frac{R^B}{R^K(\Gamma(\bar{\psi}) - \mu G(\bar{\psi})) - R^B}.$$

Substituting again:

$$\boxed{\phi_E = -\frac{R^B}{\frac{\rho(\bar{\psi})}{\beta}(\Gamma(\bar{\psi}) - \mu G(\bar{\psi})) - R^B}} \tag{D.34}$$

yields **the second equation** for the system.

We have 2 equations (D.33, D.34) and 2 unknowns ($\bar{\psi}, \phi_E$). Since we know the distribution of ψ we can solve the system. After solving the system and have a value for ψ, ϕ^E, we find R_k from ($R^K = \rho R^B$) since we know $\bar{\psi}$ and R. We have four equations, two for the bank's problem and two for the entrepreneur with all the distribution equations for ρ,ψ,$G(\psi)$ etc. These are solved by the function `SS_formal_fct.m` To find R^L we go to the only equation that has it:

$$R^L = \bar{\psi}R^K Q\frac{K}{L}.$$

We then know

$$\frac{K}{L} = \frac{1}{1 - \frac{1}{\phi^E}} = \frac{\phi^E}{\phi^E - 1}$$

D.3 Steady State of the Real Sector

Having solved for the financial sector, it's straightforward to find the steady values for the real economy. The interest rate on capital yields $R^K = r_K + 1 - \delta$. We can calculate rK since we know all the other variables. Solve for L/K:

$$\frac{H^d}{K} = \left(\frac{1}{1+g}\right)\left(\frac{u * r_K}{(1-\alpha)PWP}\right)^{\frac{1}{\alpha}}$$

From the law of motion for capital, we have

$$\frac{I}{K} = \frac{\delta + g}{1+g} \tag{D.35}$$

From the entrepreneur's consumption equation we get

$$\frac{C^E}{K} = (1-\sigma^E)(1-\xi^E)(1-\Gamma(\bar{\psi}))R_{k,ss}Q/(1+g) \tag{D.36}$$

since we know everything. The resource constraint of the economy in steady state yields $Y = C + I + G + \mu G(\bar{\psi_t})R_k QK$. Let's name g_y the steady state fraction of government spending relative to output ($\eta = G/Y$). Using the production function,

$$C = L^{1-\alpha}K^{\alpha} - \delta K - g_y L^{1-\alpha}K^{\alpha} - C^E - \mu G(\bar{\psi_t})R_k QK$$

$$\frac{Y}{K} = \frac{(H/K(1+g))^{\alpha}}{(1+F)\Delta^P}$$

Rearranging terms we get :

$$\frac{C}{K} = (1-g_y)(\frac{Y}{K})^{1-\alpha} - \frac{I}{K} - \frac{C^E}{K} - \mu G(\bar{\psi_t})R_k Q \tag{D.37}$$

$$Y/C = \frac{Y/K}{C/K}.$$

Finally we have:

$$W = \frac{P^W}{P}\alpha\left(\frac{H^d}{K(1+g)}\right)^{(\alpha-1)}$$

and

$$W^h = W * Invmarkup$$

To find C and H^d we need the labour FOC and the equations therein. We put altogether in a function `SS_formal_realsector_fct.m` with inputs W and Y/C. The system is

- U_C
- U_H
- labour FOC
- $C = \frac{Y^W/\Delta_P}{Y/C}$
- lam
- W equation solved wrt H^d

Finally, knowing L and $\frac{Y}{K}$ from the production function we find the capital. Hence, having capital, by reverse engineering we can find the values for (I, C, C^E, Y) from the equations (D.35, D.36, D.37) respectively. We know from the entrepreneur problem that

$$\frac{L}{K} = 1 - \frac{N^E}{K} = 1 - \frac{1}{\phi^E}$$

Since we know ϕ^E and the capital in steady state we now can find the value for L_t and by the same method we can find (D, N_b, N_e).

E Impulse Responses to Remaining Structural Shocks

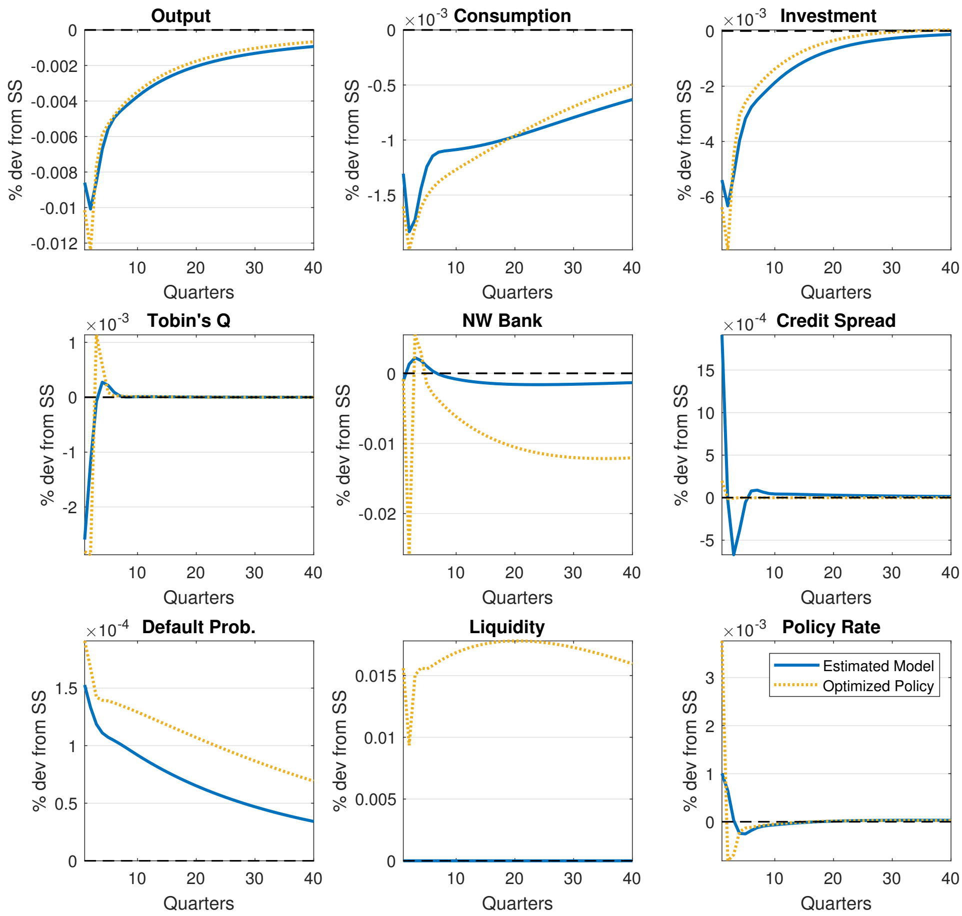

Figure 7: Impulse Responses to a Marginal Cost Shock Shock

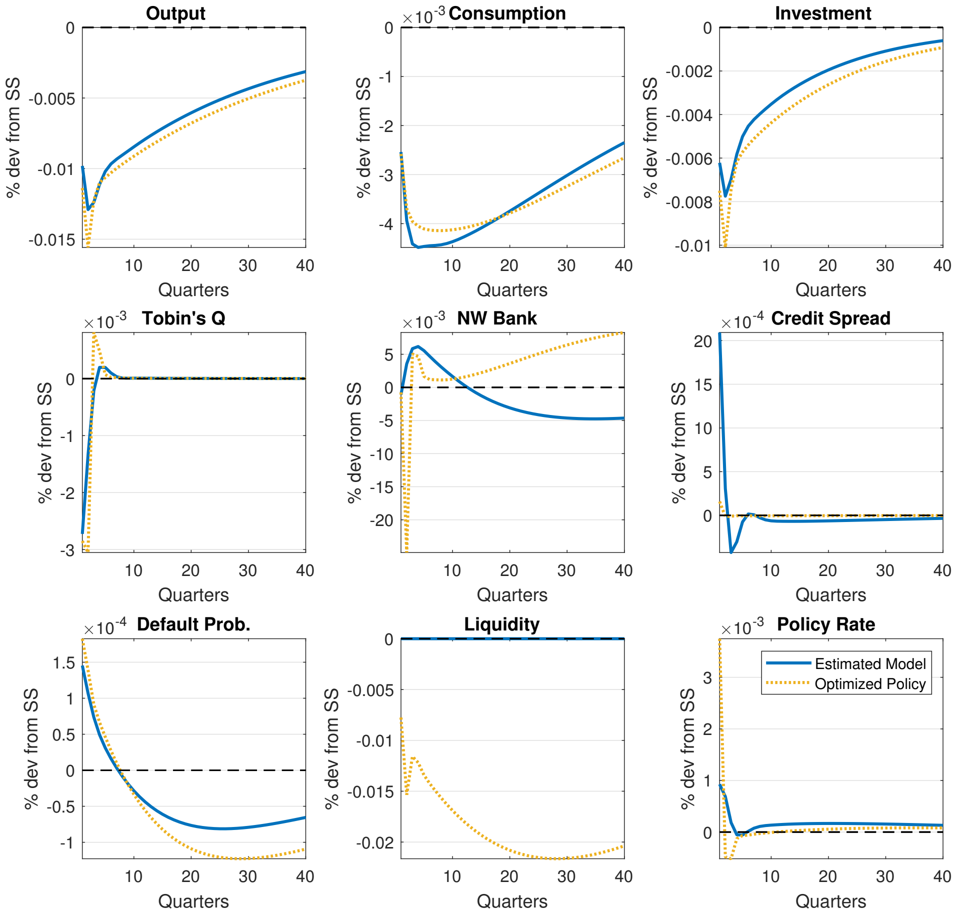

Figure 8: Impulse Responses to a Technology Shock

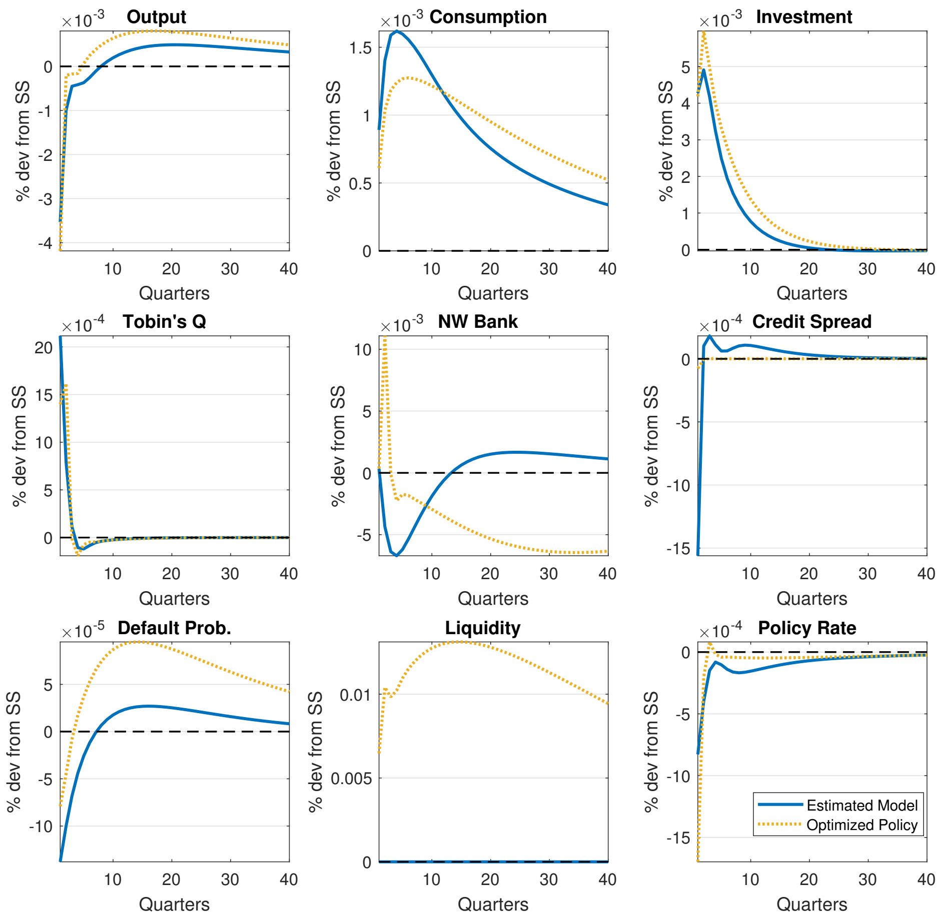

Figure 9: Impulse Responses to a Government Spending Shock

F Welfare Decomposition for Low Monitoring Ability

In the two following tables, we calculate the consumption equivalence variations for different components of the liquidity rule when the parameter ω is equal to 50%. We choose this number as a more moderate value for the monitoring parameter of the banking system. Given the low effectiveness of the liquidity in the bank's constraint, the liquidity rule becomes welfare reducing for all but one of its components in the case of no ZLB consideration.

Rule Targets	Welfare Value	Consumption Eq. Change
No Policy	-472.347	0
Spread	Indeterminacy	-
Inflation	-523.364	-13.632
Output	Indeterminacy	-
Inf. + Output	-488.806	-4.398
All	-472.171	0.047

Table 7: Welfare changes under zero lower bound. Notes: probability of hitting the zero lower bound:0.01; optimal inflation:1.008 ; $\omega = 50\%$

Rule Targets	Welfare Value	Consumption Eq. Change
No Policy	-472.199	0
Spread	Indeterminacy	-
Inflation	-519.844	-12.737
Output	Indeterminacy	-
Inf. + Output	-518.770	-12.450
All	-471.981	0.057

Table 8: Welfare changes under the assumption of no zero lower bound Notes: $\omega = 50\%$